Analytic solutions of chaotic equations

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Abstract.

Chaotic evolution equations sometimes display regular patterns, which often correspond to closed form analytic solutions. In the dissipative-dispersive Kuramoto and Sivashinsky travelling wave reduction $u(x-ct)$, $\nu u''' + bu'' + \mu u' + u^2/2 + A = 0, \nu \neq 0$, with (ν, b, μ, A) constants, such analytic solutions are known for heteroclinic solutions, but one has also observed (Toh, 1987) homoclinic solutions without corresponding analytic solutions yet. Searching for the most general analytic solution admissible by this chaotic differential equation is much more difficult than for integrable equations, because of the lack of a general method, and we will review this question. Several investigations, both analytic by the Painlevé test (Thual and Frisch, 1986) and numerical by Padé approximants (Yee, Conte, Musette, 2003) indicate its quite probable single valuedness for any (ν, b, μ, A). Moreover, Nevanlinna theory on the growth of solutions near infinity rules out (Eremenko, preprint, 2005) the possibility for this unknown closed form single valued expression to be generically meromorphic.

We reduce the search for this solution to the search for an entire function which is a deformation, yet to be found, of the entire function $\sigma(z, g_2, g_3)$ of Weierstrass violating the odd parity of the latter. The validity of this feature for the one-dimensional complex Ginzburg-Landau equation, whether cubic or quintic, will be discussed.

References.

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