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An example on the difference of two mapping-space topologies: the strong topology and the very strong topology

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Abstract

To clarify the difference of two mapping-space topologies: the strong topology and the very strong topology, A. du Plessis and H. Vosegaard wrote an example which indicates the difference of such two topologies. However we found a non-constructed part in the example. So, we construct a sufficiently concrete example, since the construction is not so trivial in the sense of technique.

1. Introduction

In this paper we consider the spaces of C^k-maps between C^k-manifolds($k=0, 1, 2, \ldots, \infty$). For such spaces, several topologies are defined: the weak topology τC^k, the strong topology $\tau W^k(k=0, 1, 2, \ldots, \infty)$ and the very strong topology τV^∞, where the notations are due to A. du Plessis and T. Wall [3]. We recall these definitions in § 2. The origin of our paper is the following lemma in [3]:

Lemma 3.4.18 Let N and P be smooth manifolds. Let U be an open subset of N, and r an integer with $0 \leq r < \infty$. For each $f \in C^r(N, P)$, there is a τW^r-neighborhood $\mathcal{U}$ of $f|_U$ in $C^r(U, P)$ such that for every $h \in \mathcal{U}$, the map $h^*: N \to P$ defined by h on U and by f outside U is C^r. Moreover, the map $h \mapsto h^*$ from $\mathcal{U}$ to $C^r(N, P)$ is $(\tau C^s, \tau W^s)$-continuous for $0 \leq s \leq r$ (where the $(\tau C^s, \tau W^s)$-continuity is the continuity between N with τC^s-topology and P with τW^s-topology).

A. du Plessis and T. Wall wrote in [3] that the above lemma should be false for the case $r=\infty$. Proposition 5.2 in [2] shows this in the category of C^∞-functions. In fact, let $N=\mathbf{R} \supset U=\mathbf{R}_+$, $P=\mathbf{R}$, $f=0$ and $\mathcal{U}$ a τW^∞-open neighborhood of $f|_U$. Then their counter-example is given by the sum $\Sigma_n \varphi_n$ if C^∞-functions $\varphi_n \in \mathcal{U}(n \in \mathrm{N})$ satisfy that $sup_{[\frac{1}{n+1}, \frac{1}{n}]} |\varphi_n^{(r+1)}| \geq 1$ for some fixed r and 0 outside $\left[\frac{1}{n+1}, \frac{1}{n}\right]$. But they did not construct such φ_n explicitly.

For this reason we want to give a more concrete counter-example in § 3. The detailed construction of the counter-example is interesting from the viewpoint of technique and is not yet appeared in the other papers. So, we believe that this result is worth publishing.

Moreover we show that if we take an arbitrary τW^∞-open neighborhood $\mathcal{U}$ for certain N, P, U and f then there always exists a C^ω-function h in $\mathcal{U}$ which cannot be extended smoothly outside U. In our construction we use a quite analytic method. For reader's convenience we deal with the behavior of some function near the origin in the appendix (§ 4).

Nevertheless J. Cerf showed the case $r=\infty$ of the above lemma for τV^∞-topology. In our previous paper [6], we have given a detailed proof for such a claim of Cerf, since he gave only a sketch of proof.

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2. Mapping-space topologies

First, let N and P be smooth manifolds, i.e., these are paracompact C^∞ manifolds. Note that if a manifold is paracompact, then it possesses a countable open basis, and it admits to possess a metric. In this section, we recall the definitions of the mapping-space topologies as given in [3].

Let $k\in\mathbf{N}$, and let D be a closed subset of N. We define

$$\mathcal{O}_{N,D}(W)=\{g\in C^\infty(N,P)\,|\,j^kg(D)\subset W\},$$

where W is an open subset in $J^k(N,P)$. Then $\{\mathcal{O}_{N,D}(W)\,|\,W\subset J^k(N,P),\ W \text{ is open}\}$ forms an open basis for a topology on $C^\infty(N,P)$, which we denote by τ_D^k. The fact that $\tau_D^k\subset\tau_D^{k+1}$ is clear. Then the topology τ_D^∞ is defined so that $\cup_k\tau_D^k$ forms its basis.

The Thom topology τC^∞ is defined so that $\cup_K\tau_K^\infty$ forms its basis, where K runs through all compact subsets of N.

The Whitney C^∞-topology (or the strong topology) τW^∞ is defined to be τ_N^∞.

Now we give the definition of the very strong topology as follows: let $\{K_\alpha\}_{\alpha\in\mathbf{N}}$ be a locally finite compact covering of N, where $\mathbf{N}$ is the set of natural numbers. Then, a basis for τV^∞ is given by all the sets of the form $\cap_\alpha\nu_\alpha$, where ν_α is a $\tau_{K_\alpha}^\infty$-open subset of $C^\infty(N,P)$. The very strong topology does not depend on the choice of the covering $\{K_\alpha\}_{\alpha\in\mathbf{N}}$ ([2], Lemma 1.1.1).

Note that when N is compact, the three topologies τC^∞, τW^∞ and τV^∞ coincide with each other.

Second, for C^r-manifolds N and P, the Thom topology $\tau C^r(0\leq r<\infty)$ and the Whitney C^r-topology (or the strong topology) $\tau W^r(r<\infty)$ on $C^r(N,P)$ are defined by using

$$\mathcal{O}^r_{N,D}(W)=\{g\in C^r(N,P)\,|\,j^rg(D)\subset W\}$$

instead of $\mathcal{O}_{N,D}(W)$. The basis of τC^r is given by $\cup_K\tau_K^r$, and τW^r is defined to be τ_N^r.

3. Detailed construction of an example

The following theorem has been given in Cerf's paper [1, I. 4.3.4.4, p. 273] with a sketch of proof and is also cited in [2, Proposition 1.2.2] without any proof. We gave a detailed proof in [6] based on the idea of the proof of Lemma 3.4.18 in [3].

Theorem 1. *Suppose N and P are smooth manifolds, U is an open subset of N and f is in $C^\infty(N, P)$. Then there is a τV^∞-neighborhood $\mathcal{U}$ of $f|_U$ in $C^\infty(U, P)$ such that the induced map $h^*: N \to P$, defined by h on U and by f outside U, is C^∞ for all $h \in \mathcal{U}$.*

The statement of Theorem 1 is false for τW^∞-case. In fact, we can construct an example as follows.

Proposition 1. *Set $N=(-\infty, 2)$ and $U=(0, 2)$. Let $f: N \to \mathbf{R}$ be the zero map. Then there is no τW^∞-open neighborhood $\mathcal{U}$ of $f|_U$ in $C^\infty(U, \mathbf{R})$ such that every analytic $h \in \mathcal{U}$ extends outside U smoothly.*

proof. Let $\mathcal{U}$ be a τW^∞-open neighborhood of $f|_U$ in $C^\infty(U, \mathbf{R})$. It is enough to show that $\mathcal{U}$ contains an analytic function h which cannot extend outside U smoothly. Since there exists an open set u in $\mathcal{U}$ and a natural number k such that $f \in u \in \tau_U^k$, we may assume that $\mathcal{U}$ is a basic τ_U^k-open set ; i.e.

$$\mathcal{U}=\{g \in C^\infty(U, \mathbf{R}) \mid j^k g(U) \subset W\}$$

for some open subset W of $J^k(U, \mathbf{R})$.

We consider a map from the jet space $J^k(U, \mathbf{R})$ to $U \times \mathbf{R}^{k+1}$ by

$$J^k(U, \mathbf{R}) \ni (x, j^k v(x)) \mapsto (x, v(x), v'(x), \cdots, v^{(k)}(x)) \in U \times \mathbf{R}^{k+1}.$$

Next we define $\mathcal{U}_m$ by

$$\mathcal{U}_m=\{(x, y) \in U \times \mathbf{R} \mid |y| < m(x)\}$$

where $m(x)=\dfrac{r(x)}{\sqrt{k+1}}$ and $r(x)$ is the supremum of radii of open balls in W centered at $(x, \underbrace{0, \cdots, 0}_{k}) \in W$.

We first construct h on $(0, 1]$. Let I be $[0, 1]$. Then there exists a C^∞-function $\sigma: I \to I$ such that

$$\begin{cases} \sigma(x)=0, & \text{on } x\in[0,\varepsilon] \\ \sigma(x)=1, & \text{on } x\in[1-\varepsilon,1] \\ \sigma'(x)\geq 0, & \text{on } x\in[0,1] \end{cases}$$

where ε is a sufficiently small positive number. We consider a finite covering of $\left[\frac{1}{2}, 1\right]$($n\in$ N) by open balls in $\mathcal{U}_m$ whose centers are in $\left[\frac{1}{2}, 1\right]$. We may assume that no balls are contained in the other ball and these balls have non-empty intersections only with neighboring balls in the covering. Assume also that the radius of the open disk containing the point $\left(\frac{1}{2}, 1\right)$ is sufficiently small. Denote the x-coordinates of the crossing points by a, b, c, $\cdots$ with $1>a>b>c>\cdots>\frac{1}{2}$, for example as in Figure 1.

Now consider a C^∞-function in this Figure 1, where $0<B\leq A$. Define $y=\tilde{f}$ on $[c, 1]$ by $y=A$ on $[a, 1]$, $y=B$ on $[c, b]$ and

$$y=\sigma\left(\frac{x-b}{a-b}\right)(A-B)+B \quad (x\in[b, a]).$$

Similarly we define $\tilde{f}$ from on $[c, 1]$ to on $\left[\frac{1}{2}, 1\right]$.

Next we repeat the same definition on $\left[\frac{1}{3}, \frac{1}{2}\right]$, gradually we get a C^∞-function on $(0, 1]$, and no generality is lost by assuming this function converges to 0 as $x\to 0$ and $0\leq\sigma'(x)\leq 1$. We call this function $\tilde{f}$ for "function like stairs". Then the next relation is satisfied on $(0, 1]$ since $\tilde{f}$ is an increasing function.

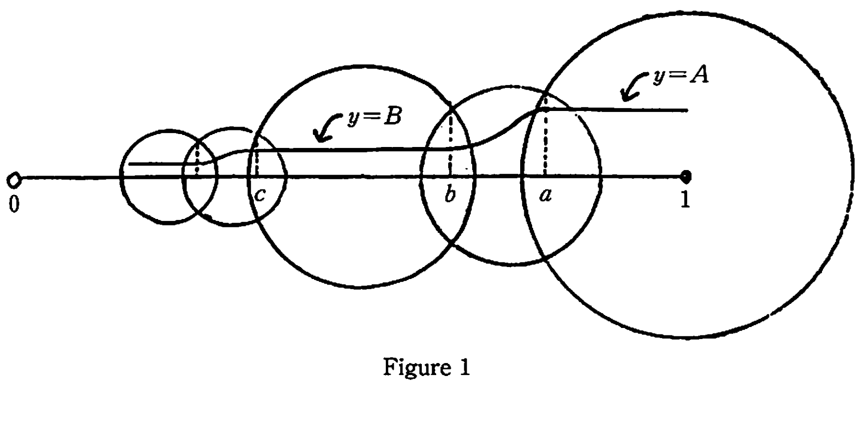

Figure 1

$$g(x)=\underbrace{\int_0^x\int_0^x\cdots\int_0^x}_{k}\bar{f}(x)dx\cdots dxdx\leq\bar{f}(x)$$

$$\cdots$$

$$\cdots$$

$$g^{(k-1)}(x)=\int_0^x\bar{f}(x)dx\leq\bar{f}(x)$$

$$0\leq g^{(k)}(x)\leq\bar{f}(x).$$

Moreover we get the following relation on $(0,1]$:

$$\left|\underbrace{\int_0^x\int_0^x\cdots\int_0^x}_{k}\bar{f}(x)\sin(\log\mathscr{H}(x))dx\cdots dxdx\right|\leq g(x)\leq\bar{f}(x)$$

$$\left|\underbrace{\int_0^x\cdots\int_0^x}_{k-1}\bar{f}(x)\sin(\log\mathscr{H}(x))dx\cdots dx\right|\leq g^{(1)}(x)\leq\bar{f}(x)$$

$$\cdots$$

$$\cdots$$

$$|\bar{f}(x)\sin(\log\mathscr{H}(x))|\leq g^{(k)}(x)\leq\bar{f}(x),$$

where we construct $\mathscr{H}(x)$ as follows:
We take a new function like stairs $H(x)$ on $(0,1]$ which satisfies $H(x)<x\bar{f}(x)$ on $(0,1]$. Then we define $\mathscr{H}(x)$ by

$$\mathscr{H}(x)=\left(\frac{1}{5}+\frac{\frac{1}{4}-\frac{1}{5}}{2}\right)H(x)+\left(\frac{\frac{1}{4}-\frac{1}{5}}{2}\right)H(x)\sin\left(\frac{1}{xH(x)}\right).$$

Now we will construct a counter-example $h(x)$ of class C^∞ on $(0,1]$. If $\bar{f}'(x)$ does not converge to 0 for $x\to 0$, then it is sufficient to define $h(x)$ by

$$h(x)=\underbrace{\int_0^x\int_0^x\cdots\int_0^x}_{k}\bar{f}(x)dx\cdots dxdx.$$

So, we may assume $\bar{f}'(x)\to 0(x\to 0)$. Then

$$h(x)=\underbrace{\int_0^x\int_0^x\cdots\int_0^x}_{k}\bar{f}(x)\sin(\log\mathscr{H}(x))dx\cdots dxdx$$

is a desired function. The fact that h belongs $\mathcal{U}$ on $(0,1]$ is explained by the previous relation, and this function cannot be extended outside $(0,1]$ from the origin. Indeed, the $(k+1)$-st derivative is

$$\begin{aligned}D^{(k+1)}h&=(\bar{f}(x)\sin(\log\mathscr{H}(x)))'\\&=\bar{f}'(x)\sin(\log\mathscr{H}(x))+\bar{f}(x)\cos(\log\mathscr{H}(x))\frac{\mathscr{H}'(x)}{\mathscr{H}(x)}.\end{aligned}$$

Since the first term converges to 0 as $x\to 0$, we consider the second term mainly. $\frac{\bar{f}(x)}{\mathscr{H}(x)}$ becomes positive infinity as $x\to 0$, because of the order for $\mathscr{H}(x)$. By Proposition 2 in the later section 4 there exists some $c>0$ such that there is some $p\in(0,\delta]$ with $\cos(\log\mathscr{H}(p))\cdot\mathscr{H}'(p)>c$ for any $\delta>0$. Hence if we extend this $h(x)$ towards outside U then the $(k+1)$-st derivative of $h(x)$ does not become continuous at the origin, i.e., this is a C^∞ counter-example on $(0,1]$ for Theorem 1 with τW^∞ instead of τV^∞.

We can construct a C^∞ counter-example $\hat{h}(x)$ on $U=(0,2)$ in $\mathcal{U}$ by using a function $\varepsilon(x)$ in $C^\infty((0,1],\mathbf{R})$. It is ε on $\left(0,\frac{1}{2}\right]$ and $\sigma(x)$ on $\left[\frac{1}{2},1\right]$ where $\varepsilon>0$ is sufficiently small, and $\sigma(x)$ is a bump function in $C^\infty\left(\left[\frac{1}{2},1\right],\mathbf{R}\right)$ such as $\sigma(x)=\varepsilon$ near $\left(\frac{1}{2},0\right)$, $\sigma(x)=0$ near $(1,0)$ where $\|j^k\sigma(x)\|$ is sufficiently small on $\left[\frac{1}{2},1\right]$. Then $\hat{h}(x)$ is defined by $h(x)\varepsilon(x)$ on $(0,1]$ and 0 on $[1,2)$. Next we construct an analytic counter-example based on this $\hat{h}(x)$.

$$\mathscr{A}=\left\{g\in C^\infty(U,\mathbf{R})\,|\,|D^{(i)}(\hat{h}(x)-g(x))|<x^2,\quad g\in\mathcal{U},\quad 0\le i\le k+1\right\}$$

is an open neighborhood of $\hat{h}(x)$ in τW^∞. For a preparation, we quote the following lemma in [5] (p. 65).

Lemma 1. (Grauert-Remmert) *Let N and P be C^ω manifolds. Then $C^\omega(N,P)$ is dense in $C^\infty(N,P)$ with respect to τW^∞-topology.*

From this Lemma 1, $\mathscr{A}$ contains an analytic function $\bar{g}(x)$. This $\bar{g}(x)$ is our analytic counter-example. In fact, $\bar{g}(x)\in\mathcal{U}$ and the $(k+1)$-st derivative shakes near 0 in the above sense and $\bar{g}(x)$ cannot be extended outside U smoothly.

Remark 1. *In general, the above $h(x)$ on $(0, 1]$ cannot be extended on $(0, 2)$ keeping the condition that $\hat{h}(x)$ coincides with $h(x)$ on $(0, 1]$.*

4. Appendix

We use the xy-plane $\mathbf{R}\times\mathbf{R}$. We consider a C^∞-function $y=H(x)$ on $(0, 1]$ which satisfies the following conditions: $H(x)>0$ on $(0, 1]$, $\lim_{x\to+0}H(x)=0$, $0\leq\frac{d}{dx}H(x)\leq 1$.

Define $y=\mathscr{H}(x)$ on $(0, 1]$ by

$$\mathscr{H}(x)=aH(x)+bH(x)\sin\left(\frac{1}{xH(x)}\right)$$

where $a=\beta+\frac{\alpha-\beta}{2}$, $b=\frac{\alpha-\beta}{2}$ with $\alpha>\beta>0$, for example $\alpha=\frac{1}{4}$ and $\beta=\frac{1}{5}$ as in § 3.

Proposition 2. *There exists a constant $c>0$ such that for any $\varepsilon>0$ there is some $p\in(0, \varepsilon]$ which satisfies*

$$|\cos(\log\mathscr{H}(p))\mathscr{H}'(p)|\geq c.$$

proof. (outline) First we calculate the derivative of $\mathscr{H}(x)$,

$$\mathscr{H}'(x)=aH'(x)+bH'(x)\sin\left(\frac{1}{xH(x)}\right)+bH(x)\cos\left(\frac{1}{xH(x)}\right)\times\left(-\frac{H(x)+xH'(x)}{(xH(x))^2}\right).$$

Then the third term is

$$(\bigstar)\quad \left(\frac{1}{x}\right)\left(b\cos\left(\frac{1}{xH(x)}\right)\right)\left(-\frac{1}{x}-\frac{H'(x)}{H(x)}\right)$$

whence $\left(\frac{1}{x}\right)\left(-\frac{1}{x}-\frac{H'(x)}{H(x)}\right)\to-\infty$ (for $x\to+0$) bacause $\frac{H'(x)}{H(x)}\geq 0$. From the condition $0\leq H'(x)\leq 1$, we notice the fact that the behavior of $\mathscr{H}'(x)$ is dominated by the third term $(\bigstar)$ when x is close to 0. We define points $(x_i, 0)(0<x_i\leq 1, i\in\mathrm{N})$ as follows:
$\cos(\log\beta H(x_i))=\frac{1}{\sqrt{2}}$, $\log\beta H(x_i)-\log\beta H(x_{i+1})=2\pi$, $\frac{d}{dx}\{\cos(\log\beta H(x_1))\}\leq 0$, and if x_i lies in a continuous interval then we choose the point of the right edge of that interval.

On the interval $[x_{i+1}, x_i]$, an interval $[l_i, x_i]$ is defined by

$$\left\{x\in[x_{i+1}, x_i]\,|\cos(\log\beta H(x))\geq\frac{1}{\sqrt{2}}\right\}.$$

From the condition $0\leq H'(x)\leq 1$, i.e., $0\leq\frac{d}{dx}(\beta H(x))\leq\beta$, we get

$$(A)\quad \frac{1}{4}\cdot\frac{1}{\beta}(\beta H(x_i)-\beta H(x_{i+1}))<x_i-l_i<x_i-x_{i+1}.$$

Step 1 :

We define the period of $\mathscr{H}(x)$ in $[x_{i+1}, x_i]$ by $\max\left\{\text{the period of } \sin\left(\frac{1}{xH(x)}\right)\middle| x\in[x_{i+1}, x_i]\right\}$, and we estimate this period of $\mathscr{H}(x)$ in $[x_{i+1}, x_i]$. Since

$$\frac{2\pi}{\left(\frac{1}{xH(x)}\right)'}=2\pi\frac{-(xH(x))^2}{H(x)+xH'(x)} \text{ and } 0\leq H'(x)\leq 1,$$

$$\{\text{the period of } \mathscr{H}(x) \text{ in } [x_{i+1}, x_i]\}<\frac{2\pi(x_iH(x_i))^2}{H(x_i)}=2\pi x_i^2H(x_i)$$

where the right term of the above inequality is the case of $H'(x_i)=0$. Then we will compare the degree of becoming smaller between the left term of (A) $\frac{1}{4}(H(x_i)-H(x_{i+1}))$ and the period of $\mathscr{H}(x)$

$$\frac{\frac{1}{4}(H(x_i)-H(x_{i+1}))}{2\pi x_i^2H(x_i)}=\frac{1}{8\pi}\left(\frac{1}{x_i^2}\left(1-\frac{H(x_{i+1})}{H(x_i)}\right)\right).$$

By using the relation $H(x_{i+1})=\frac{H(x_i)}{e^{2\pi}}$, we get

$$\frac{1}{8\pi}\left(\frac{1}{x_i^2}\left(1-\frac{H(x_{i+1})}{H(x_i)}\right)\right)=\frac{1}{8\pi}\left(\frac{1}{x_i^2}\left(1-\frac{1}{e^{2\pi}}\right)\right)\to+\infty \quad (\text{for } i\to+\infty).$$

Therefore the order of becoming smaller of $\mathscr{H}(x)$'s period is larger than the order of becoming smaller of $\frac{1}{4}(H(x_i)-H(x_{i+1}))$ and x_i-x_{i+1}.

Step 2 :

Next we compare the degree of becoming smaller between the amplitude of $\mathscr{H}(x)$ at x, i.e., $\alpha H(x)-\beta H(x)$ and $\beta H(x_i)-\beta H(x_{i+1})$. The amplitude of $\mathscr{H}(x)$ at x_{i+1} is $\alpha H(x_{i+1})-\beta H(x_{i+1})$ $=(\alpha-\beta)H(x_{i+1})$, then

$$\frac{(\alpha-\beta)H(x_{i+1})}{\beta H(x_i)-\beta H(x_{i+1})}=\frac{\alpha-\beta}{\beta}\cdot\frac{H(x_{i+1})}{e^{2\pi}H(x_{i+1})-H(x_{i+1})}=\frac{\alpha-\beta}{\beta}\cdot\frac{1}{e^{2\pi}-1}$$

where the first equality comes from the same argument in Step 1. Similarly, the amplitude of $\mathscr{H}(x)$ at x_i is $(\alpha-\beta)H(x_i)$, and

$$\frac{(\alpha-\beta)H(x_i)}{\beta H(x_i)-\beta H(x_{i+1})}=\frac{\alpha-\beta}{\beta}\cdot\frac{1}{1-\frac{1}{e^{2\pi}}}.$$

Hence

$$\frac{\alpha-\beta}{\beta}\cdot\frac{1}{e^{2\pi}-1}\leq\frac{\alpha H(x)-\beta H(x)}{\beta H(x_i)-\beta H(x_{i+1})}\leq\frac{\alpha-\beta}{\beta}\cdot\frac{1}{1-\frac{1}{e^{2\pi}}},\quad x\in[x_{i+1},x_i]$$

for arbitrary i.

Final Step :

Then we divide $(0, 1]$ whenever the period of $\mathscr{H}(x)$ proceeds 2π on $(0, 1]$ with x's approaching to 0, i.e., by every proceeding 2π of $\frac{1}{xH(x)}$, and denote these dividing points by $t_k(k\in\mathbf{N})$ from x_1 towards 0, then let Y_k and $Z_k(k\in\mathbf{N})$ be

$$Y_k=\left\{\mathscr{H}(x)\middle|\,|\mathscr{H}'(x)|\geq\frac{1}{2},\quad x\in[t_{k+1},k_t]\right\}$$

$$Z_k=\{\mathscr{H}(x)\,|\,x\in[t_{k+1},k_t]\}.$$

Moreover put $\widetilde{Y_k}$ as the longest interval in Y_k. Then we can find a sufficiently large $N_1(\in\mathbf{N})$ such that if $k\geq N_1$ then

$$\frac{\text{length}\,\widetilde{Y_k}}{\text{length}Z_k}\geq\frac{3}{4}.$$

This reason is explained by the first expression (★). In addition,

$$\widetilde{Y_k}\cap\widetilde{Y_{k+1}}\neq\phi$$

holds, because the order of the becoming smaller of the period of $\mathscr{H}(x)$ is sufficiently small for that of the amplitude $(\alpha-\beta)H(x)$, $\beta H(x_i)-\beta H(x_{i+1})$ and x_i-x_{i+1} by Step 1, 2 with the inequality (A). Therefore if we take a sufficiently large $N_2(\in\mathbf{N})$ then for any point $y\in[\beta H(l_i), \beta H(x_i)](i\geq N_2)$, we can find a point p such as

$$y=\mathscr{H}(p),\quad |\mathscr{H}'(p)|\geq\frac{1}{2},\quad p\in[x_{i+1},x_i].$$

This concludes our proof.

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