ON A MODIFICATION OF SCHEFFÉ'S AND URA'S ANALYSIS FOR PAIRED COMPARISONS*

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1. Introduction. H. Scheffé[1] and S. Ura[2] treated the analysis of variance of paired comparisons. In Scheffé's model, each judge judges only once one ordered pair of two brands, whence a number of judges are indispensably for us, and also, in his model, he includes into the error term the difference due to the individual variation among the judges.

In laboratory experiments or in certain kinds of delicate taste testing experiments, it may happen that the paired comparison experiments must be achieved by several expert judges. And, in such cases, it is necessary for each judge to test all ordered pairs. Therefore, Scheffé's model seems to be inadequate for us.

While, the aforementioned defects caused by the application of Scheffé's model in special kinds of experiments, are considerably eliminated in Ura's model.

That is: (1) As each judge compares only once each of all the possible ordered pairs formed from m kinds of food, so are the numbers of judges equal to the numbers of the times of repetition for each ordered pair only necessary. (2) He introduces the term representing the differences among individual sensations of individual preferences as the interaction between foods and judges in his model. (3) Moreover, in his model, he concludes, into the error term, the order effect in Scheffé's model which means the difference due to order of presentation in the preference for food T_i over food T_j, and further he introduces the term of the individual variation for the order effect.

In certain kinds of our taste testing experiments, it seems most likely to introduce the term of (2), and, in addition, the order effect in (3) in the case of Scheffé's model, while it seems unnecessary to introduce the term of the individual variation for the order effect in (3).

Thus, hereafter, we shall adopt a modification of Scheffé's and Ura's models as described in section 3.

2. The Method of Experiment. Suppose there are m foods $T_1, T_2, \cdots\cdots, T_m$, to be compared. All the $2M$ possible ordered pairs are formed, where $M=\binom{m}{2}=\frac{m(m-1)}{2}$, that is,

$$
\begin{gathered}
(T_1, T_2), (T_1, T_3), \cdots\cdots\cdots, (T_1, T_m),\\
(T_2, T_1), (T_2, T_3), \cdots\cdots\cdots, (T_2, T_m),\\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots,\\
(T_m, T_1), (T_m, T_2), \cdots\cdots, (T_m, T_{m-1}),
\end{gathered}
$$

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where (T_i, T_j) means the ordered pair performed from two foods T_i, T_j. Then r judges compare once each of these pairs respectively and report the results in the 7-point scoring system, that is, in comparing of (T_i, T_j) in the order (i, j),

if he prefers T_i to T_j strongly then his score is 3,
if he prefers T_i to T_j moderately then his score is 2,
if he prefers T_i to T_j slightly then his score is 1,
if no preference then his score is 0,
if he prefers T_j to T_i slightly then his score is -1,
if he prefers T_j to T_i moderately then his score is -2,
if he prefers T_j to T_i strongly then his score is -3.

Each judge compares these $2M$ pairs in a random order. In order to diminish the weariness of their senses of taste, it is necessary to prepare suitably a fixed rest among the experiments.

3. Model of Data. Let us denote by x_{ijk} the score which the judge 0_k assigns on an ordered pair (T_i, T_j). We assume the following mathematical model:

$$(1)\quad x_{ijk}=(\alpha_i+\alpha_{ik})-(\alpha_j+\alpha_{jk})+\gamma_{ij}+\overbrace{\delta'_{ij}+\delta_0}^{\delta_{ij}}+\varepsilon_{ijk}\quad (x_{iik}=0),$$

where the parameters have the following means respectively:

α_i characterizes an object T_i and satisfies

$$\sum_{i=1}^{m}\alpha_i=0\ (i=1, 2, \cdots, m),$$

α_{ik} is an individual difference of preference for an object T_i and satisfies

$$\sum_{i=1}^{m}\alpha_{ik}=0,\ \sum_{k=1}^{r}\alpha_{ik}=0\ (i=1, 2, \cdots, m; k=1, 2, \cdots, r),$$

γ_{ij} represents a deviation[1] from subtractivity and satisfies the relations

$$\gamma_{ij}=-\gamma_{ji},\ \sum_{j=1}^{m}\gamma_{ij}=0,\ \gamma_{ii}=0\ (i, j=1, 2, \cdots, m),$$

δ_0 is an average order effect, that is,

$$\delta_0=\sum_j\sum_i\delta_{ij}/2M,$$

$2\delta_{ij}$ is a difference due to order of presentation in the mean preference for T_i over T_j, and satisfies

$$\delta_{ij}=\delta_{ji}\ (i, j=1, 2, \cdots, m),$$

δ'_{ij} is an deviation of δ_{ij} from the average order effect δ_0, and satisfies

$$\sum_j\sum_i\delta'_{ij}=0,$$

ε_{ijk} is an observational error which distributes independently each other and normaly with the mean 0 and the variance σ^2 $(i, j=1, 2, \cdots, m;\ k=1, 2, \cdots, r)$.

4. Estimation of Parameters. For simplicity, we use the following notations:

$$x_{ij\cdot}=\sum_{k=1}^{m} x_{ijk},\quad x_{i\cdot k}=\sum_{j=1}^{m} x_{ijk},\quad x_{\cdot jk}=\sum_{i=1}^{m} x_{ijk},$$

(2)

$$x_{i\cdot\cdot}=\sum_{j=1}^{m}\sum_{k=1}^{r} x_{ijk},\quad x_{\cdot j\cdot}=\sum_{i=1}^{m}\sum_{k=1}^{r} x_{ijk},\quad x_{\cdots}=\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{r} x_{ijk}.$$

Using these notations, we can obtain the estimates of parameters as follows:

$$\hat{\alpha}_i=\frac{1}{2mr}(x_{i\cdot\cdot}-x_{\cdot i\cdot})=\frac{1}{2}(\bar{x}_{i\cdot\cdot}-\bar{x}_{\cdot i\cdot}),$$

(3)

$$\hat{\alpha}_{ik}=\frac{1}{2m}(x_{i\cdot k}-x_{\cdot ik})-\hat{\alpha}_i=\frac{1}{2}(\bar{x}_{i\cdot k}-\bar{x}_{\cdot ik})-\frac{1}{2}(\bar{x}_{i\cdot\cdot}-\bar{x}_{\cdot i\cdot}),$$

$$\hat{\gamma}_{ij}=\frac{1}{2r}(x_{ij\cdot}-x_{ji\cdot})-(\hat{\alpha}_i-\hat{\alpha}_j)=\hat{\pi}_{ij}-(\hat{\alpha}_i-\hat{\alpha}_j)$$

$$=\frac{1}{2}(\bar{x}_{ij\cdot}-\bar{x}_{ji\cdot})-\left\{\frac{1}{2}(\bar{x}_{i\cdot\cdot}-\bar{x}_{\cdot i\cdot})-\frac{1}{2}(\bar{x}_{j\cdot\cdot}-\bar{x}_{\cdot j\cdot})\right\},$$

where π_{ij} is a average preference for i over j that is averaged over the two orders, and $\pi_{ij}=-\pi_{ji}$, and $\hat{\pi}_{ij}$ is a estimate of π_{ij},

$$\hat{\delta}_0=\frac{1}{m(m-1)r}x_{\cdots}=\bar{x}_{\cdots},$$

$$\hat{\delta}_{ij}=\frac{1}{2r}(x_{ij\cdot}+x_{ji\cdot})=\frac{1}{2}(\bar{x}_{ij\cdot}+\bar{x}_{ji\cdot}),$$

$$\hat{\delta}'_{ij}=\frac{1}{2r}(x_{ij\cdot}+x_{ji\cdot})-\frac{1}{m(m-1)r}x_{\cdots}=\frac{1}{2}(\bar{x}_{ij\cdot}+\bar{x}_{ji\cdot})-\bar{x}_{\cdots}$$

$$=\hat{\delta}_{ij}-\hat{\delta}_0,$$

where

$$\bar{x}_{\cdots}=\frac{x_{\cdots}}{m(m-1)r},\qquad \bar{x}_{ij\cdot}=\frac{1}{r}\sum_{k=1}^{r} x_{ijk},$$

$$\bar{x}_{i\cdot k}=\frac{1}{m}\sum_{j=1}^{m} x_{ijk},\qquad \bar{x}_{\cdot jk}=\frac{1}{m}\sum_{i=1}^{m} x_{ijk},$$

$$\bar{x}_{i\cdot\cdot}=\frac{1}{mr}\sum_{j=1}^{m}\sum_{k=1}^{r} x_{ijk},\qquad \bar{x}_{\cdot j\cdot}=\frac{1}{mr}\sum_{i=1}^{m}\sum_{k=1}^{r} x_{ijk}.$$

5. Analysis of Variance. The total sum of square S_t is divided into six components S_α, S_{α_k}, S_γ, S_{δ_0}, $S_{\delta'}$, S_e, and these sum of squares are statistically independent, where they are respectively sum of squares due to main effects, interactions between main effects and individual preference, deviations from subtractivity, average order effect, deviations from average order effect, error.

Each sum of square are computable as following:

$$
\begin{aligned}
S_t=&\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{r} x_{ijk}^2=\sum_i\sum_j\sum_k\Bigg[\{x_{ijk}-\Big(\frac{\bar{x}_{i\cdot k}-\bar{x}_{\cdot ik}}{2}-\frac{\bar{x}_{i\cdot\cdot}-\bar{x}_{\cdot i\cdot}}{2}\\
&-\frac{\bar{x}_{j\cdot k}-\bar{x}_{\cdot jk}}{2}+\frac{\bar{x}_{j\cdot\cdot}-\bar{x}_{\cdot j\cdot}}{2}\Big)-\bar{x}_{ij\cdot}\}+\Big\{\frac{\bar{x}_{ij\cdot}+\bar{x}_{ji\cdot}}{2}-\bar{x}_{\cdots}\Big\}+\{\bar{x}_{\cdots}\}\\
&+\Big\{\Big(\frac{\bar{x}_{i\cdot k}-\bar{x}_{\cdot ik}}{2}-\frac{\bar{x}_{i\cdot\cdot}-\bar{x}_{\cdot i\cdot}}{2}\Big)-\Big(\frac{\bar{x}_{j\cdot k}-\bar{x}_{\cdot jk}}{2}-\frac{\bar{x}_{j\cdot\cdot}-\bar{x}_{\cdot j\cdot}}{2}\Big)\Big\}\\
&+\Big\{\frac{\bar{x}_{ij\cdot}-\bar{x}_{ji\cdot}}{2}-\Big(\frac{\bar{x}_{i\cdot\cdot}-\bar{x}_{\cdot i\cdot}}{2}-\frac{\bar{x}_{j\cdot\cdot}-\bar{x}_{\cdot j\cdot}}{2}\Big)\Big\}+\Big\{\frac{\bar{x}_{i\cdot\cdot}-\bar{x}_{\cdot i\cdot}}{2}-\frac{\bar{x}_{j\cdot\cdot}-\bar{x}_{\cdot j\cdot}}{2}\Big\}\Bigg]^2,
\end{aligned}
$$

$$
S_\alpha=r\sum_{i=1}^{m}\sum_{j=1}^{m}\Big(\frac{\bar{x}_{i\cdot\cdot}-\bar{x}_{\cdot i\cdot}}{2}-\frac{\bar{x}_{j\cdot\cdot}-\bar{x}_{\cdot j\cdot}}{2}\Big)^2=\frac{1}{2mr}\sum_i(x_{i\cdot\cdot}-x_{\cdot i\cdot})^2,
$$

$$
\begin{aligned}
S_{\alpha_k}=&\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{r}\Big\{\Big(\frac{\bar{x}_{i\cdot k}-\bar{x}_{\cdot ik}}{2}-\frac{\bar{x}_{i\cdot\cdot}-\bar{x}_{\cdot i\cdot}}{2}\Big)-\Big(\frac{\bar{x}_{j\cdot k}-\bar{x}_{\cdot jk}}{2}-\frac{\bar{x}_{j\cdot\cdot}-\bar{x}_{\cdot j\cdot}}{2}\Big)\Big\}^2\\
=&\frac{1}{2m}\sum_i\sum_k(x_{i\cdot k}-x_{\cdot ik})^2-\underbrace{\frac{1}{2mr}\sum_i(x_{i\cdot\cdot}-x_{\cdot i\cdot})^2}_{S_\alpha},
\end{aligned}
$$

$$
\begin{aligned}
S_\gamma=&\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{r}\Big\{\frac{\bar{x}_{ij\cdot}-\bar{x}_{ji\cdot}}{2}-\Big(\frac{\bar{x}_{i\cdot\cdot}-\bar{x}_{\cdot i\cdot}}{2}-\frac{\bar{x}_{j\cdot\cdot}-\bar{x}_{\cdot j\cdot}}{2}\Big)\Big\}^2\\
=&\frac{1}{2r}\sum_{i<j}{}'(x_{ij\cdot}-x_{ji\cdot})^2-\underbrace{\frac{1}{2mr}\sum_i(x_{i\cdot\cdot}-x_{\cdot i\cdot})^2}_{S_\alpha},
\end{aligned}
$$

$$
S_{\delta_0}=\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{r}\{\bar{x}_{\cdots}\}^2=\Big(\frac{x_{\cdots}^2}{m(m-1)r},
$$

$$
S_{\delta'}=\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{r}\Big\{\frac{\bar{x}_{ij\cdot}+\bar{x}_{ji\cdot}}{2}-\bar{x}_{\cdots}\Big\}^2=\frac{1}{2r}\sum_{i<j}{}'(x_{ij\cdot}+x_{ji\cdot})^2-\underbrace{\frac{x_{\cdots}^2}{m(m-1)r}}_{S_{\delta_0}},
$$

$$
(S_\delta=\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{r}\Big\{\frac{\bar{x}_{ij\cdot}+\bar{x}_{ji\cdot}}{2}\Big\}^2=\frac{1}{2r}\sum_{i<j}{}'(x_{ij\cdot}+x_{ji\cdot})^2=S_{\delta_0}+S_{\delta'},
$$

where S_δ is a sum of square due to the order effect for i over j)

$$
\begin{aligned}
S_e=&\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{r}\Big[x_{ijk}-\frac{\bar{x}_{i\cdot k}-\bar{x}_{\cdot ik}}{2}+\frac{\bar{x}_{j\cdot k}-\bar{x}_{\cdot jk}}{2}+\frac{\bar{x}_{i\cdot\cdot}-\bar{x}_{\cdot i\cdot}}{2}-\frac{\bar{x}_{j\cdot\cdot}-\bar{x}_{\cdot j\cdot}}{2}-\bar{x}_{ij\cdot}\Big]^2\\
=&\sum_i\sum_j\sum_k\Big[x_{ijk}-\frac{x_{i\cdot k}-x_{\cdot ik}}{2m}+\frac{x_{j\cdot k}-x_{\cdot jk}}{2m}+\frac{x_{i\cdot\cdot}-x_{\cdot i\cdot}}{2mr}-\frac{x_{j\cdot\cdot}-x_{\cdot j\cdot}}{2mr}-\frac{x_{ij\cdot}}{r}\Big]^2\\
=&S_t-(S_\alpha+S_\gamma+S_{\alpha_k}+S_\delta),
\end{aligned}
$$

where summation $\sum_{i<j}{}'$ denotes the sum over all the possible ordered pairs (i, j), $i<j$.

We show the analysis of variance based on our model in Table 1.

Table 1.

Source	Sum of Squares	Degrees of Freedom	Mean Square
Main Effects	S_α	$m-1$	$S_\alpha/(m-1)$
Main Effects × Judges	S_{α_k}	$(m-1)(r-1)$	$S_{\alpha_k}/(m-1)(r-1)$
Deviation from Subtractivity	S_γ	$\frac{(m-1)(m-2)}{2}$	$S_\gamma\Big/\left(\frac{(m-1)(m-2)}{2}\right)$
Order Effects	S_δ	$\frac{m(m-1)}{2}$	$S_\delta\Big/\left(\frac{m(m-1)}{2}\right)$
[Average Order Effect	[S_{δ_0}	[1	[S_{δ_0}
Deviation from Average Order Effect]	$S_{\delta'}$]	$\frac{m^2-m-2}{2}$]	$S_{\delta'}\Big/\left(\frac{m^2-m-2}{2}\right)$]
Error	S_e	$(m-1)^2(r-1)$	
Total	S_t	$m(m-1)r$	

6. Numerical Example. The foods are four kinds of sausages made from some fish meat which contain respectively Ribotide and Monosodium Glutamate with the

Table 2. Scores by Judges $0_1, \cdots, 0_6$

j \ i	T_1	T_2	T_3	T_4	$x_{\cdot j1}$
T_1	*	3	1	−2	2
T_2	1	*	−1	0	0
T_3	−1	−1	*	1	−1
T_4	0	2	1	*	3
$x_{i\cdot 1}$	0	4	1	−1	4
$x_{\cdot i1}$	2	0	−1	3	
$(x_{i\cdot 1}-x_{\cdot i1})$	−2	4	2	−4	
$(\quad)^2$	4	16	4	16	40

j \ i	T_1	T_2	T_3	T_4	$x_{\cdot j2}$
T_1	*	1	−1	1	1
T_2	0	*	1	−2	−1
T_3	0	2	*	−1	1
T_4	−2	3	0	*	1
$x_{i\cdot 2}$	−2	6	0	−2	2
$x_{\cdot i2}$	1	−1	1	1	
$(x_{i\cdot 2}-x_{\cdot i2})$	−3	7	−1	−3	
$(\quad)^2$	9	49	1	9	68

j \ i	T_1	T_2	T_3	T_4	$x_{\cdot j3}$
T_1	*	0	−1	−1	−2
T_2	1	*	1	−1	1
T_3	1	1	*	0	2
T_4	2	2	0	*	4
$x_{i\cdot 3}$	4	3	0	−2	5
$x_{\cdot i3}$	−2	1	2	4	
$(x_{i\cdot 3}-x_{\cdot i3})$	6	2	−2	−6	
$(\quad)^2$	36	4	4	36	80

j \ i	T_1	T_2	T_3	T_4	$x_{\cdot j4}$
T_1	*	1	−1	−1	−1
T_2	1	*	−1	−3	−3
T_3	−1	2	*	1	2
T_4	0	1	1	*	2
$x_{i\cdot 4}$	0	4	−1	−3	0
$x_{\cdot i4}$	−1	−3	2	2	
$(x_{i\cdot 4}-x_{\cdot i4})$	1	7	−3	−5	
$(\quad)^2$	1	49	9	25	84

j \ i	T_1	T_2	T_3	T_4	$x_{\cdot j5}$
T_1	*	1	−1	−2	−2
T_2	1	*	0	−1	0
T_3	1	1	*	−1	1
T_4	2	2	1	*	5
$x_{i\cdot 5}$	4	4	0	−4	4
$x_{\cdot i5}$	−2	0	1	5	
$(x_{i\cdot 5}-x_{\cdot i5})$	6	4	−1	−9	
$(\quad)^2$	36	16	1	81	134

j \ i	T_1	T_2	T_3	T_4	$x_{\cdot j6}$
T_1	*	1	−1	0	0
T_2	0	*	1	−1	0
T_3	−2	0	*	1	−1
T_4	1	2	2	*	5
$x_{i\cdot 6}$	−1	3	2	0	4
$x_{\cdot i6}$	0	0	−1	5	
$(x_{i\cdot 6}-x_{\cdot i6})$	−1	3	3	−5	
$(\quad)^2$	1	9	9	25	44

concentration combination ($\alpha\%$, $\beta\%$), ($3\alpha\%$, $\beta\%$), ($5\alpha\%$, $\beta\%$), (0, $1.5\beta\%$), and let us denote by the signs T_1, T_2, T_3, T_4 respectively for these four foods.

The judges are six expert panels $0_1, 0_2, \cdots, 0_6$ which were chosen by means of the method of reference (3) from the students of Kumamoto Women's University.

The data in Table 2 were obtained in a taste testing experiment performed by the above judges $0_1, 0_2, \cdots, 0_6$ ($r=6$) for T_1, T_2, T_3, T_4 ($m=4$).

Table 3. Sum of Scores

j \ i	T_1	T_2	T_3	T_4	$x_{\cdot j \cdot}$
T_1	*	7	-4	-5	-2
T_2	4	*	1	-8	-3
T_3	-2	5	*	1	4
T_4	3	12	5	*	20
$x_{i\cdot\cdot}$	5	24	2	-12	19
$x_{\cdot i\cdot}$	-2	-3	4	20	
$(x_{i\cdot\cdot}-x_{\cdot i\cdot})$	7	27	-2	-32	
$(\quad)^2$	49	729	4	1,024	1,806

Table 4. $\hat{\alpha}_i$ and $\hat{\alpha}_{ik}$

i	$\hat{\alpha}_i=\frac{x_{i\cdot\cdot}-x_{\cdot i\cdot}}{48}$	$\frac{x_{i\cdot 1}-x_{\cdot i1}}{8}$	$\hat{\alpha}_{i1}$	$\frac{x_{i\cdot 2}-x_{\cdot i2}}{8}$	$\hat{\alpha}_{i2}$	$\frac{x_{i\cdot 3}-x_{\cdot i3}}{8}$	$\hat{\alpha}_{i3}$
1	.1458	—.2500	—.3958	—.3750	—.5208	.7500	.6042
2	.5625	.5000	—.0625	.8750	.3125	.2500	—.3125
3	—.0417	.2500	.2917	—.1250	—.0833	—.2500	—.2083
4	—.6667	—.5000	.1667	—.3750	.2917	—.7500	—.0833

i	$\frac{x_{i\cdot 4}-x_{\cdot i4}}{8}$	$\hat{\alpha}_{i4}$	$\frac{x_{i\cdot 5}-x_{\cdot i5}}{8}$	$\hat{\alpha}_{i5}$	$\frac{x_{i\cdot 6}-x_{\cdot i6}}{8}$	$\hat{\alpha}_{i6}$
1	.1250	—.0208	.7500	.6042	—.1250	—.2708
2	.8750	.3125	.5000	—.0625	.3750	—.1875
3	—.3750	—.3333	— .1250	—.0833	.3750	.4167
4	—.6250	.0417	—1.1250	—.4583	—.6250	.0417

Table 5. $\hat{\gamma}_{ij}$

	$(x_{ij\cdot}-x_{ji\cdot})$				$\frac{(x_{ij\cdot}-x_{ji\cdot})}{12}$				$(x_{ij\cdot}-x_{ji\cdot})^2$			
i \ j	1	2	3	4	1	2	3	4	1	2	3	4
1	*	-3	2	8	*	—.2500	.1667	.6667	*	9	4	64
2	*	*	4	20	*	*	.3333	1.6667	*	*	16	400
3	*	*	*	4	*	*	*	.3333	*	*	*	16
4	*	*	*	*	*	*	*	*	*	*	*	*

	$\hat{\alpha}_i-\hat{\alpha}_j$				$\hat{\gamma}_{ij}=\frac{x_{ij\cdot}-x_{ji\cdot}}{12}-(\hat{\alpha}_i-\hat{\alpha}_j)$			
i \ j	1	2	3	4	1	2	3	4
1	*	−.4167	.1875	.8125	*	.1667	−.0208	−.1458
2	*	*	.6042	1.2292	*	*	−.2709	.4375
3	*	*	*	.6250	*	*	*	−.2917
4	*	*	*	*	*	*	*	*

Table 6. $\hat{\delta}_{ij}$, $\hat{\delta}_0$, $\hat{\delta}'_{ij}$

	$(x_{ij\cdot}+x_{ji\cdot})$				$\hat{\delta}_{ij}=(x_{ij\cdot}+x_{ji\cdot})/12$				$(x_{ij\cdot}+x_{ji\cdot})^2$				$\hat{\delta}'_{ij}=\hat{\delta}_{ij}-\hat{\delta}_0$			
i \ j	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1	*	11	−6	−2	*	.9167	−.5000	−.1667	*	121	36	4	*	.6528	−.7639	−.4306
2	*	*	6	4	*	*	.5000	.3333	*	*	36	16	*	*	.2361	.0694
3	*	*	*	6	*	*	*	.5000	*	*	*	36	*	*	*	.2361
4	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*

$$\hat{\delta}_0=\frac{x_{\cdots}}{72}=\frac{19}{72}=.2639$$

Table 7. Analysis of Variance

Modification					Ura's Model				
Source	S. of S.	D. of F.	M. S.	Result	Source	S. of S.	D. of F.	M. S.	Result
Main Effects	$S_\alpha=37.63$	3	12.54	* *	Main Effects	$S_\alpha=37.63$	3	12.54	* *
Main Effects × Judges	$S_{\alpha_k}=18.62$	15	1.24	—	Main Effects × Judges	$S_{\alpha_k}=18.62$	15	1.24	—
Deviation from Subtractivity	$S_\gamma=4.79$	3	1.60	—	Deviation from Subtractivity	$S_\gamma=4.79$	3	1.60	—
Order Effects	$S_\delta=20.75$	6	3.46	* *	Order Effects × Judges	$S_\delta=6.42$	6	1.07	
[Average Order Effect	[$S_{\delta_0}=5.01$	[1	[5.01	[*	[Average Order Effect	[$S_{\delta_0}=5.01$	[1	[5.01	—
Deviation from Average Order Effect]	$S_{\delta'}=15.74$]	5]	3.15]	*]	Deviation from Average Order Effect]	$S_{\delta'}=1.41$]	5]	0.28]	
Error	$S_e=43.21$	45	0.96	—	Error	$S_e=57.54$	45	1.28	—
Total	$S_t=125$	72	—	—	Total	$S_t=125$	72	—	—

Conclusion: (1) The null hypothesis H: $\alpha_i=0$ (for all i) can be rejected with $\alpha=0.01$ in both analysis of variance based on each model. (2) In the analysis of variance based on the modification model, the hypothesis H: average order effect $\delta_0=0$ can be rejected and the hypothesis that all δ_{ij} are equal each other can be also rejected ($\alpha=0.05$). (3) In the analysis of variance based on Ura's model, the null hypothesis H: $\delta_0=0$ can be not rejected and moreover the significant difference between individual order effects δ_k is not recognized, because the value of the error sum of square is larger than the value of the case of the modification.

Whence it seems that we could explain more faithfully this kind of the experimental data by means of the modification model.

7. Some Remarks. (1) In our model, we introduce four terms representing the following effects respectively: (a) the main effect, (b) the deviation from subtractivity for i over j, (c) the individual variation of the main effect, (d) the order effect for i over j.

While, in our model, as the error term, we treat the following variations: (a′) the individual variation of the deviation from subtractivity, (b′) the individual variation of the order effect, (c′) the individual preference of each judge, (d′) the other experimental error.

(2) Taking into consideration of the observed scores, we are unnecessary to adopt the 7-point scoring system essentially in numerical example of section 6. Therefore, we would like here to adopt the 5-point scoring system.

Moreover, in the above example, we have not tried to test the hypothesis of the homogeneity of variance with respect to the data.

In conclusion, the author wishes to express his heartiest thanks to Doctor A. Kudo for his kind advice and valuable suggestions in connection with this work.

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Reference

1) H. Scheffé, "*An Analysis of Variance for Paired Comparisons*," J. Amer. Statist. Assoc., **47**, 381 (1952).

2) S. Ura, "*Statistical Methods in Sensory Tests, Chapter 3, A Modification of Scheffé's Analysis of Paired Comparisons*," Ph. D. Dissertation (1961).

3) S. Kinoshita, "*Process of Making Most Excellent Soup-stock in Special Consideration of Taste Examination. I*," J. Kumamoto Women's Univ., **14**, 15 (1962).