CORRECTION

ON THE THEORY OF DIFFERENTIAL EQUATIONS IN COORDINATED SPACES

Mituo INABA

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To the beginning of theorem 3, add the sentence "Let E be a coordinated space with a topology $\mathcal{J}_1$, which has either (i) the properties (P) and (AK), or (ii) a fundamental system of neighborhoods of the origin consisting of products of intervals."

Delete the phrase "for a topology $\mathcal{J}_1$" at the end of the first sentence of theorem 3.

The proof of theorem 3 then should be corrected as follows:

PROOF. Case (i). Since $x(t)$ and $f(t, x)$ are continuous for the topology $\mathcal{J}_1$, the function $x'(t)=f(t, x(t))$, derivative for the topology $\mathcal{J}_2$, is continuous for the topology $\mathcal{J}_1$. For the sequence $\{x'_n(t);\ n=1, 2, \cdots\}$, $x'^{[n]}(t)$ is continuous and converges to $x'(t)$ (derivative for the topology $\mathcal{J}_2$) for the topology $\mathcal{J}_1$ uniformly on the interval I' ([5], lemma 1). As is shown by a well-known process, the functions $x(t)$ is derivable (for the topology $\mathcal{J}_1$) and its derivative is equal to $x'(t)$, the limit function of $x'^{[n]}(t)$, which proves the theorem.

Case (ii). Let $x_{(n)}$ denote the projection $x^{[n]}-x^{[n-1]}$ for $n>1$ and $x^{[1]}$ for $n=1$. Then, we have, for the topology $\mathcal{J}_2$,

$$\begin{aligned}\frac{x_{(n)}(t+h)-x_{(n)}(t)}{h} &= x'_{(n)}(t+\theta_n h)\\ &= f_{(n)}(t+\theta_n h,\ x(t+\theta_n h)) \qquad (0<\theta_n<1)\end{aligned}$$

for t and $t+h \in I'$. By virtue of the continuity of $x(t)$ and $f(t, x)$ for the topology $\mathcal{J}_1$, to any neighborhood U of the origin for the topology $\mathcal{J}_1$, there exists a positive number δ, such that we have

$$f(t+h, x(t+h))-f(t, x(t)) \in U$$

for $|h|<\delta$, and consequently,

$$\begin{aligned}\frac{x_{(n)}(t+h)-x_{(n)}(t)}{h} &- f_{(n)}(t,\ x(t))\\ &= f_{(n)}(t+\theta_n h,\ x(t+\theta_n h))-f_{(n)}(t,\ x(t)) \in U.\end{aligned}$$

The property of the neighborhood U implies the relation:

$$\frac{x(t+h)-x(t)}{h}-f(t,\ x(t)) \in U$$

for $|h|<\delta$, which shows $x'(t)=f(t, x(t))$ for the topology $\mathcal{T}_1$.

Replace the phrase "for a topology $\mathcal{T}$" at the end of the first sentence of the corollary of theorem 3, by the phrase "for a topology $\mathcal{T}$, which is specified as $\mathcal{T}_1$ in the theorem."

Department of Mathematics,
Faculty of Science,
Kumamoto University