RIMS workshop

Modular Forms and Automorphic Representations

(Titles and Abstracts)

Feb. 2 (Mon)

13:30-14:30 **Shuichi Hayashida** (Joetsu University of Education)

Title: Generalized Maass relations and lifts.

Abstract: In this talk I would like to explain two things.
(1) Lifts from two elliptic modular forms to Siegel modular forms of half-integral weight of even degrees. These lifts are constructed by a composition of the Duke-Imamoglu-Ikeda lifts, map of the Fourier-Jacobi expansion, and the Eichler-Zagier-Ibukiyama correspondence. Under the assumption that the constructed Siegel modular form is not identically zero, we obtain the fact that the constructed form is a Hecke eigenform. And the Zhuravlev L-function of such a form is a product of L-functions of elliptic modular forms. A generalized Maass relation of half-integral weight plays an important rule in the proof.
(2) The explicit expression of the spinor L-function of some Miyawak-Ikeda lifts of odd degrees. A generalized Maass relation of integral weight gives such explicit expression of the spinor L-function of Miyawaki-Ikeda lifts. This is a product of symmetric tensor L-functions of elliptic modular forms. The case of degree three has already been obtained by B.Heim.

14:45-15:45 **Tomoya Kiyuna** (Kyushu University)

Title: Kaneko-Zagier type differential equation for Jacobi forms.

Abstract: We introduce a certain fourth order partial differential equation related to Jacobi forms. We show several properties of the equation and give explicit Jacobi form solutions.

16:00-17:00 **Tomoyoshi Ibukiyama** (Osaka University)

Title: Three theorems on Jacobi forms of general degree.

Abstract: I talk on three new theorems related to Jacobi forms of general degree stated as follows:
(1) Taylor coefficients of any holomorphic Jacobi forms of matrix index is essentially vector valued Siegel modular forms.
(2) The module of holomorphic Jacobi forms of degree two of level 2 is explicitly determined.
(3) Holomorphic or skew holomophic vector valued Jacobi forms of index one of general degree of level N is isomorphic to vector valued Siegel modular forms of half integral weight of level $4N$ in the plus space.

Feb. 3 (Tue)

9:45-10:45 **Tomomi Ozawa** (Tohoku University)

Title: Constant terms of Eisenstein series over a totally real field.

Abstract: In this talk, we compute constant terms of Eisenstein series defined over a totally real field at all cusps. M. Ohta defined and computed congruence modules related to Eisenstein series defined over the field $\mathbb{Q}$ of rational numbers in a paper published in 2003. His theory of congruence modules has been applied to several important problems in number theory, particularly in Iwasawa theory. In his computation, the constant terms of Eisenstein series over $\mathbb{Q}$ at all the equivalence classes of cusps are necessary. We explicitly describe the constant terms of Eisenstein series over a general totally real field in terms of special values of Hecke L-functions, at each equivalence class of cusps.

11:00-12:00 **Kaoru Okada** (Ritsumeikan University)

Title: Hecke algebras for Hilbert modular forms and ideal class groups.

Abstract: Using twisting operators defined by characters of order two of $Cl^+(F)$, we present a connection between $Cl^+(F)/(Cl^+(F))^2$ and quadratic subextensions of a certain subalgebra in the Hecke algebra acting on spaces of Hilbert modular forms over F. Here $Cl^+(F)$ is the ideal class group of F in the narrow sense. In addition, we give examples related to this result. This is a joint work with Yoshio Hiraoka.

13:30-14:30 **Soma Purkait** (Kyushu University)

Title: Hecke algebras, new vectors and characterization of the new space.

Abstract: Let p be a prime. Let $K_0(p^n)$ be the subgroup of $\mathrm{GL}_2(\mathbb{Z}_p)$ consisting of matrices with lower left entry in $p^n\mathbb{Z}_p$. We shall consider the Hecke algebra of $\mathrm{GL}_2(\mathbb{Q}_p)$ with respect to $K_0(p^n)$ and its subalgebra that is supported on $\mathrm{GL}_2(\mathbb{Z}_p)$ and describe them using generators and relations. This will allow us to explicitly describe the representations of $\mathrm{GL}_2(\mathbb{Z}_p)$ having a $K_0(p^n)$ fixed vector. We will translate this information to the classical setting which will lead us to characterize the space of newforms for $\Gamma_0(M)$ as a common eigenspace of certain Hecke operators which depends on prime divisors of M.

14:45-15:45 **Kohta Gejima** (Osaka University)

Title: Shintani functions on $SL(2,\mathbf{C})$ and Heun's differential equations.

Abstract: We study Shintani functions attached to non-unitary principal series representations of $\mathrm{SL}(2,\mathbf{C})$. Let $\pi_{\nu,m}$ be a non-unitary principal series representation with $(m+1)$-dimensional minimal SU(2)-type.By using Zuckerman's technique, we give an inductive formula which gives a relation between the Shintani functions attached to $\pi_{\nu,m}$ and those attached to $\pi_{\nu+1,m+1}$.From this, we obtain explicit formulas of Shintani functions attached to $\pi_{\nu,m}(|m| = 1, 2)$. By definition, the Shintani function satisfies a system of differential

equations. For $|m| = 2$, the system gives us an interesting example of Heun's differential equations.

16:00-17:00 **Yoshinori Mizuno** (Tokushima University)

Title: Dirichlet series of 3 variables and Koecher-Maass series of non-holomorphic Siegel-Eisenstein series.

Abstract: We study certain Dirichlet series of 3 variables, whose Dirichlet coefficients involve the product of quadratic L-functions. We give meromorphic continuations and a vector functional equation of the Dirichlet series. Our main motivation comes from a certain Dirichlet series (called by Koecher-Maass series) associated with non-holomorphic Siegel-Eisenstein series of even degree. It is classically known that the Koecher-Maass series for holomorphic Siegel modular forms can be continued to the whole complex plane and has a functional equation. While, the approaches applied in this holomorphic case due to Maass, Koecher and Arakawa have not been worked out successfully yet. As a typical example of non-holomorphic Siegel modular forms, we are concerned with non-holomorphic Siegel-Eisenstein series. The Koecher-Maass series associated with the non-holomorphic Siegel-Eisenstein series whose degree larger than two was studied by Arakawa, Suzuki, Ibukiyama-Katsurada, Sato-Ueno. In addition, we treated the case of degree 2 with positive definite Fourier coefficients, and the case of degree 2 and weight 2 with indefinite Fourier coefficients. In these cases, the associated Koecher-Maass series has a meromorphic continuation and a functional equation. The main goal of this talk is to define a Koecher-Maass series of indefinite Fourier coefficients of the degree 2 non-holomorphic Siegel-Eisenstein series reasonably (without any restriction about the weight) in the sense that the associated Koecher-Maass series has a meromorphic continuation and a functional equation.

Feb. 4(Wed)

10:00-11:00 **Kaoru Hiraga** (Kyoto University)

Title: Endoscopic classification of representations: A survey.
Abstract: In this survey talk, I will explain the endoscopic classification of local and global representations of Sp(n) and O(n) announced by J. Arthur. I will explain the role of the standard and twisted endoscopies in the Langlands functoriality and explain the classification of representations using the endoscopies.

11:15-12:15 **Shunsuke Yamana** (Kyushu University)

Title: Local symmetric square L-functions and trilinear forms.

Abstract: I will develop a local theory of symmetric square L-functions for general linear groups and prove a certain characterization of a pole of symmetric square L-factors of square-integrable representations, a uniqueness of certain trilinear forms and nonexistence of Whittaker models of higher exceptional representations.

14:00-15:00 **Miki Hirano** (Ehime University), **Taku Ishii** (Seikei University), **Tadashi Miyazaki** (Kitasato University)

Title: Archimedean zeta integrals for GL(3) $\times$ GL(2).

Abstract: We compute archimedean zeta integrals for Whittaker functions on GL(3) and GL(2). In particular, we give Whittaker functions explicitly, for which the archimedean zeta integrals coincide with the associated L-factors.

15:15-16:15 **Takayuki Oda** (University of Tokyo)

Title: Is there brave new world of modular symbols? Some evidences in the literature.

Abstract: Matsushima isomorphism means that cohomology classes of arithmetic quotients of symmetric spaces are represented by harmonic automorphic forms. To construct (dual-?) harmonic representatives in their duals, i.e. in homology classeis, is a big problem. However we know this only for elliptic modular curves and Hilbert modular varieties. Let me review some more "examples " among the attempts for the geometry in the sense of Klein-Cartan.

Feb. 5(Thu)

9:45-10:45 **Masataka Chida** (Kyoto University, Hakubi center)

Title: Regulators and special values of Rankin-Selberg L-functions.

Abstract: By Beilinson's conjecture, it is expected that special values of L-functions for motives at non-critical points are related to regulators from motivic cohomology to Deligne cohomology for algebraic varieties. In this talk, we will construct an element in the motivic cohomology of the product of two Kuga-Sato varieties using Beilinson's Eisenstein symbol and describe an explicit formula between the regulator and special values of Rankin-Selberg L-functions for two modular forms at some non-critical points. This is a joint work with François Brunault.

11:00-12:00 **Kimball Martin** (University of Oklahoma/Osaka City University, JSPS fellow)

Title: A Chebotarevesque principle for modularity.

Abstract: Let K/k be a Galois extension of number fields. The Chebotarev density theorem implies that Galois representations over K are determined by their behavior at split places. Ramakrishnan showed an analogue of this for automorphic representations. Consequently, to show a Galois representation and automorphic representation correspond, it should suffice to show they correspond at (almost all) split places. I will discuss joint work with Ramakrishnan on this for 2-dimensional Artin representations.

13:30-14:30 **Takeo Okazaki** (Nara Women's University)

Title: Newforms for GU(2,2)

Abstract: Let E be a quadratic extension of a nonarchimedean filed F, and GU(2,2) be the quasi-split unitary similitude group attached to E/F. Let π be an irreducible, admissible, generic representation of GU(2, 2), and $L(s, \pi)$ be its twisted exterior square L-function. We show $L(s, \pi)$ has a functional equation, and π has a Whittaker function W fixed by an arithmetic subgoup determined by the epsilon factor of the functional equation. The subspace fixed by this subgroup is one-dimensional, and a zeta integral of W is equal to $L(s, \pi)$, so, we call W the new form of π.

14:45-15:45 **Ameya Pitale** (University of Oklahoma)

Title: Lowest weight modules for Sp(4,R) and nearly holomorphic Siegel modular forms

Abstract: In this talk, we discuss the lowest weight modules for the Hermitian symmetric pair (G,K), where G=Sp(4, R) and K is its maximal compact subgroup. In particular, we write down explicit differential operators that navigate all the highest weight vectors of such a module starting from the unique lowest-weight vector. We study these operators from the representation-theoretic and classical viewpoints. We show that the automorphic forms on G that correspond to the highest weight vectors are exactly those that arise from nearly holomorphic vector-valued Siegel modular forms of degree 2.

By explicating the algebraic structure of the relevant space of automorphic forms, we are able to prove a structure theorem for the space of nearly holomorphic vector-valued Siegel modular forms of (arbitrary) weight $\det^l \mathrm{sym}^m$ with respect to an arbitrary congruence subgroup of Sp(4, Q). We show that the cuspidal part of this space is the direct sum of subspaces obtained by applying explicit differential operators to holomorphic vector-valued cusp forms of weight $\det^{l'} \mathrm{sym}^{m'}$ with (l', m') varying over a certain set. The structure theorem for the space of all modular forms is similar, except that we may now have an additional component coming from certain nearly holomorphic forms of weight $\det^3 \mathrm{sym}^{m'}$ that cannot be obtained from holomorphic forms.

As an application of our structure theorem, we prove several arithmetic results concerning nearly holomorphic modular forms that improve previously known results in that direction. This is joint work with Abhishek Saha and Ralf Schmidt.

16:00-17:00 **Siegfried Böcherer** (Universität Mannheim)

Title: On quasimodular Siegel modular forms.

Abstract: By definition, quasimodular forms are constant terms of nearly holomorphic modular forms, e.g. certain holomorphic derivatives of holomorphic modular forms are quasimodular. We show that such quasimodular forms are p-adic modular forms. Our main tools are Shimura's structure theory of nearly holomorphic modular forms and our work with Nagaoka on derivatives of modular forms as p-adic modular forms. Our approach to such

questions is different from (and more elementary than) T.Ichikawa's geometric one.

Feb. 6(Fri)

10:00-11:00 **Henry Kim** (University of Toronto)

Title: Ikeda type lift for the exceptional group of type E_7.

Abstract: Ikeda constructed a cusp form on Sp_{2n} (rank $2n$) from Hecke eigenform on the upper half plane which has been conjectured by Duke-Imamoglu and Ibukiyama. We give a similar construction on the 27-dimensional exceptional tube domain where the exceptional group of type E_7 acts. This is a joint work with T. Yamauchi.

11:15-12:15 **Tamotsu Ikeda** (Kyoto University), **Hidenori Katsurada** (Muroran Institute of Technology)

Title: On the Gross-Keating invariant of a quadratic form and its application.

Abstract: An explicit formula of the Siegel series of a half-integral symmetric matrix over $\mathbb{Z}_p$ is known, but its appearance for $p \neq 2$ and $p = 2$ are different. Gross and Keating introduced the Gross-Keating invariants to remedy this problem, and obtained an unified formula for $n \leq 3$. In this talk, we establish some basic properties of the Gross-Keating invariants of a non-degenerate quadratic form defined over the integer ring of a non-archimedean local field of characteristic zero. We will also discuss its application to the theory of the Siegel series.