Magnetic Fields from Cosmological Perturbations

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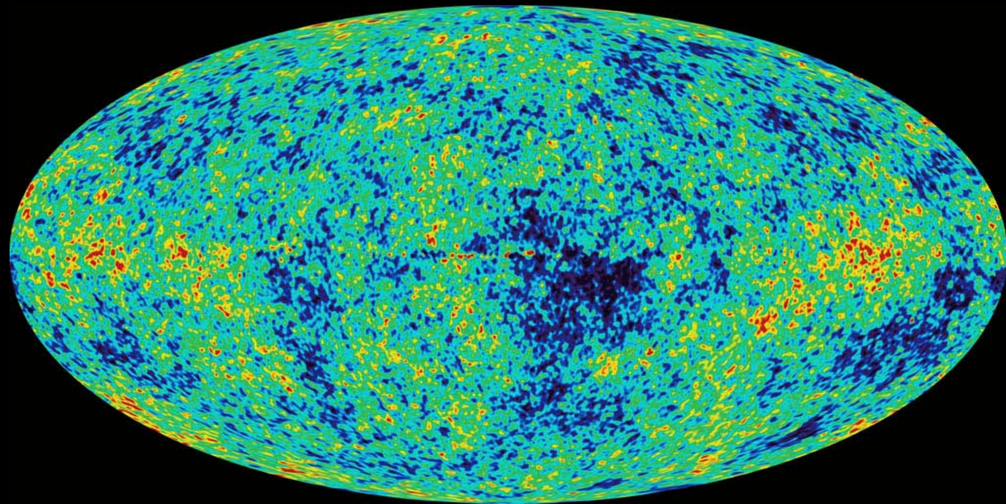
1. Introduction
2. Electromagnetic properties of the early universe
3. Magnetogenesis & tight coupling approximation

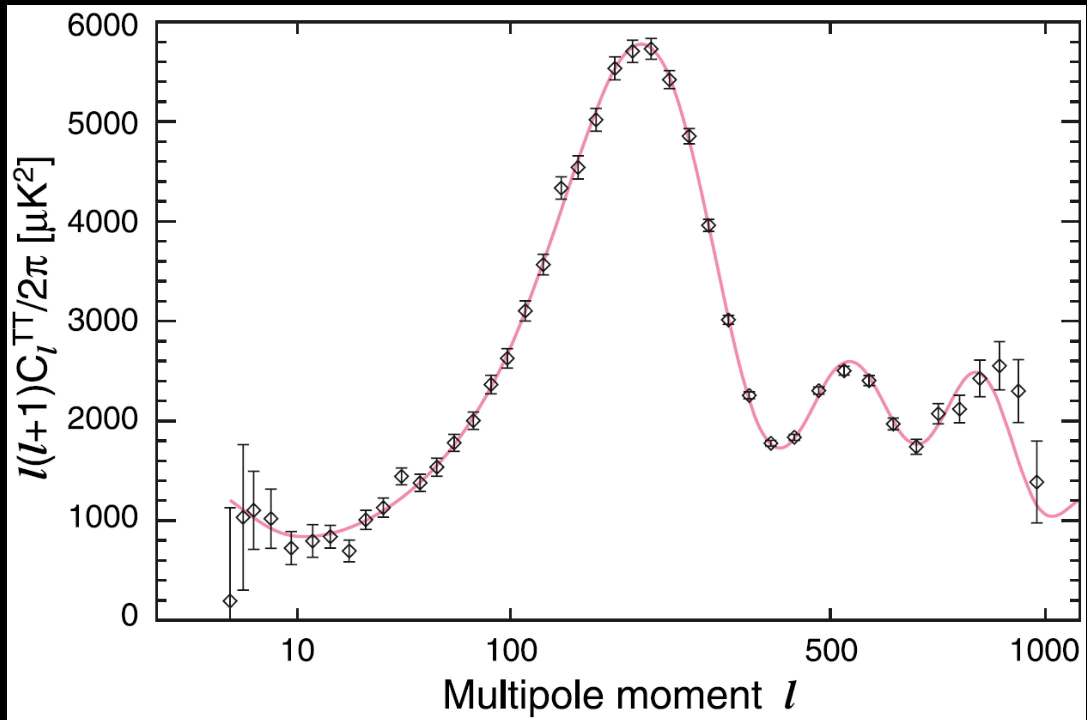
1. Introduction

primordial fluctuations

observation
- CMB
- galaxy distribution

theory
- inflation
  (initial condition)
- cosmological
  perturbation theory
  (linear)





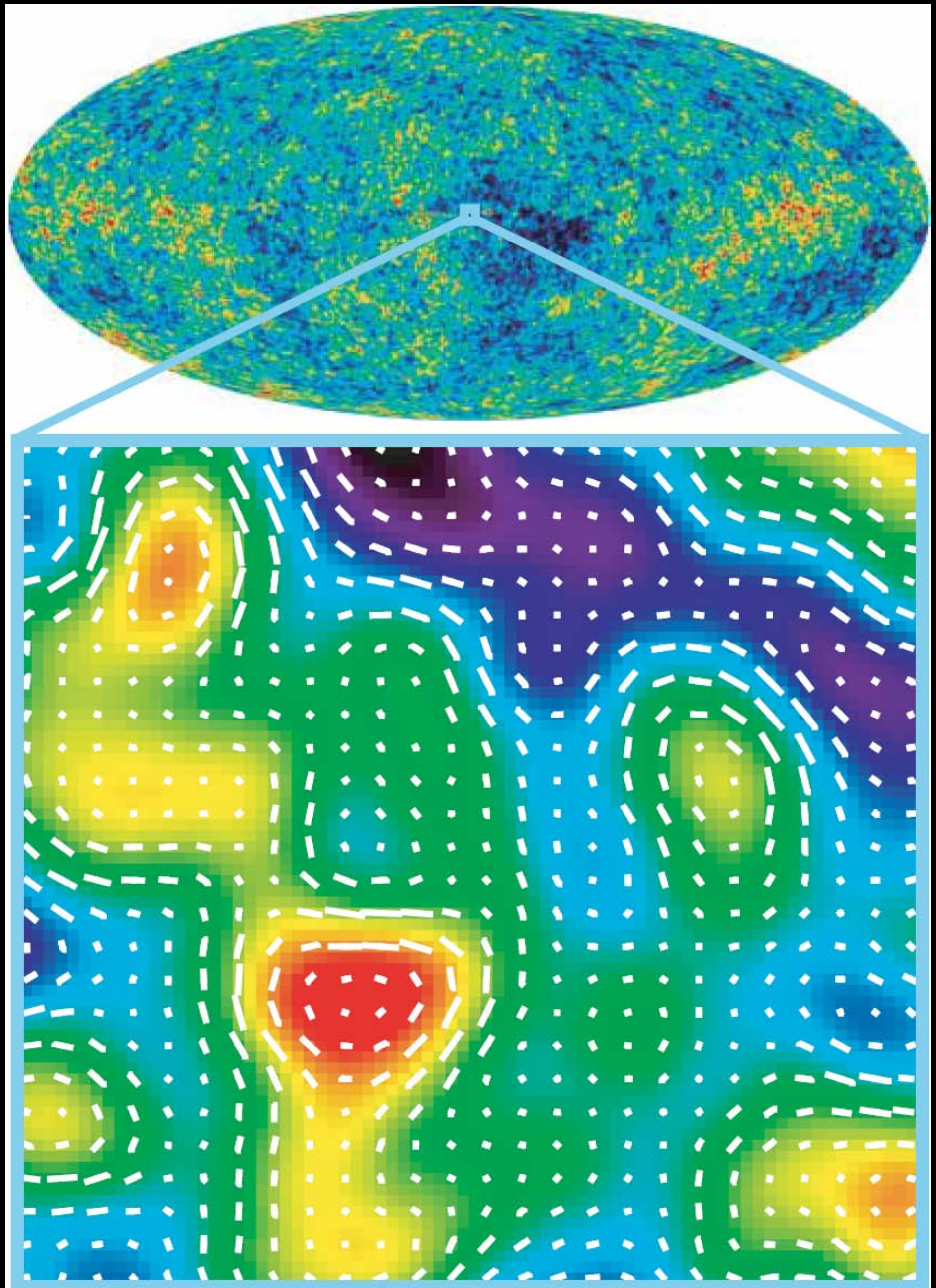
magnetogenesis

magnetogenesis from cosmological perturbations before recombination

- Hogan (2000)
- Berezhiani & Dolgov (2004)
- Matarrese et al. (2005)
- Gopal & Sethi (2005)
- KT et al. (2005, 2006, 2007, 2008)
- Siegel & Fry (2006)
- Hollenstein et al. (2008)
- Maeda et al. (2009)

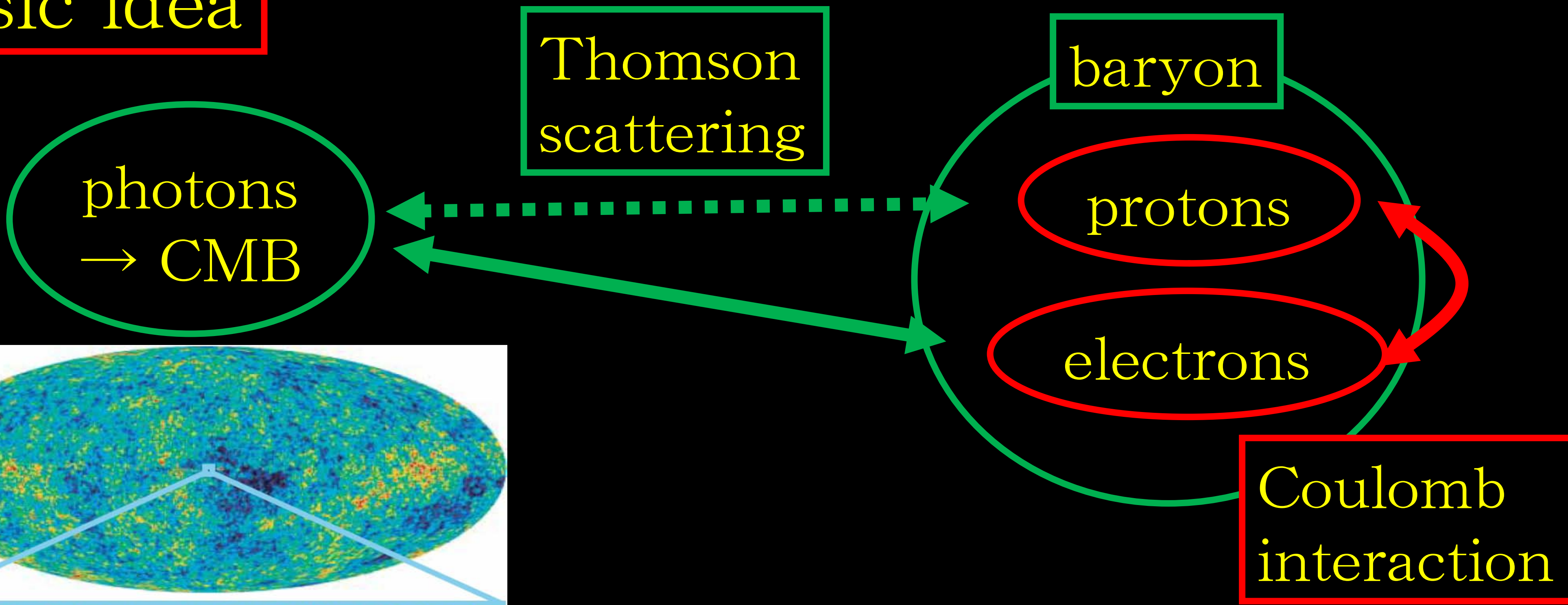
based on

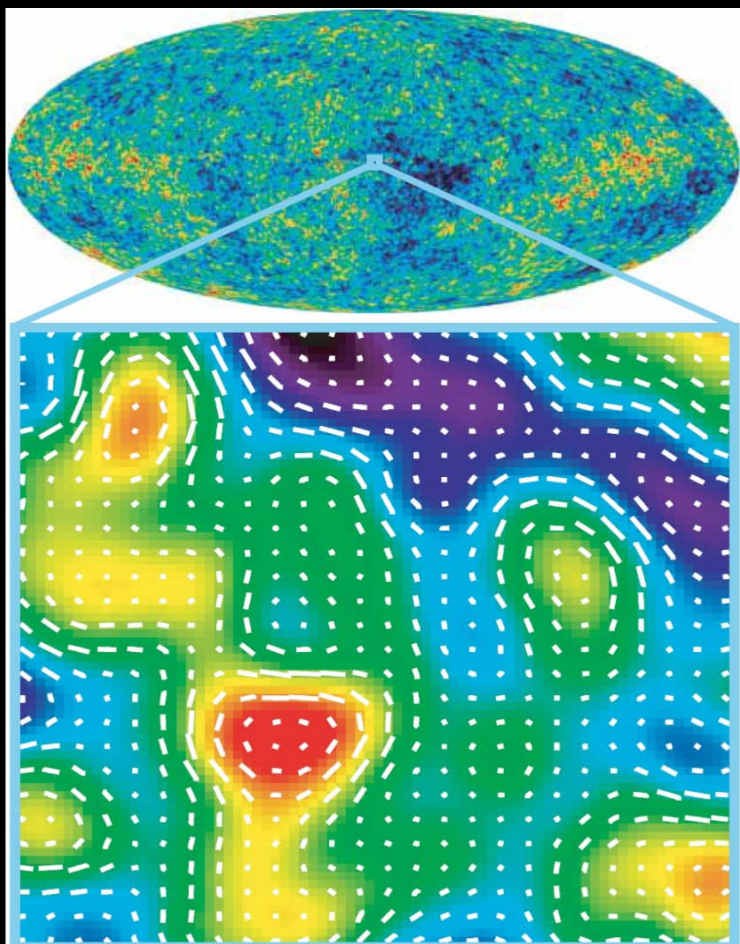
- cosmological perturbation theory (nonlinear)
- observational facts
- no physical assumption



KT et al., Science 311 (2006) 827

basic idea





Thomson scattering
→ deviation in motion
due to mass difference
→ net electric charge density
and electric current
→ magnetic fields

extensions to the conventional formalism

What do we need for magnetogenesis?

$$\partial_t \vec{B} = -\nabla \times \vec{E}$$

electric field and its rotation

electric field

- Conventionally, baryons
- Separate treatment of p and e is necessary.

rotational part (Roy's talk)

- No rotational part at the linear order
- generated by nonlinear effect

  Linear order is sufficient for CMB

  but insufficient for B.

Two extensions are needed for magnetogenesis.

this talk

understanding the physics of magnetogenesis from cosmological perturbations

- electromagnetic properties of the early universe
  - solve Maxwell and Ohm
  - Newtonian
  - neglect anisotropic stress
- tight coupling approximation
  - express B by familiar quantities (δ_γ)
  - compare with another approach

2. Electromagnetic properties of the early universe

KT, Ichiki & Sugiyama, PRD 77 (2008) 124028

EOMs

(Newtonian) EOMs for photons, protons & electrons, neglecting the anisotropic stress of photons.

$$\frac{4}{3}\rho_\gamma\left[\partial_t\vec{v}_\gamma + H\vec{v}_\gamma + \left(\vec{v}_\gamma\cdot\frac{\nabla}{a}\right)\vec{v}_\gamma\right]$$

Thomson

$$= -\frac{1}{3a}\nabla\rho_\gamma - \frac{m_e^2}{m_p^2}\sigma_T n_p \rho_\gamma(\vec{v}_\gamma - \vec{v}_p) - \sigma_T n_e \rho_\gamma(\vec{v}_\gamma - \vec{v}_e) - \frac{4\rho_\gamma}{3a}\nabla\Phi$$

$$m_p n_p\left[\partial_t\vec{v}_p + H\vec{v}_p + \left(\vec{v}_p\cdot\frac{\nabla}{a}\right)\vec{v}_p\right]$$

$$= e n_p(\vec{E} + \vec{v}_p\times\vec{B}) - e^2 n_p n_e \eta(\vec{v}_p - \vec{v}_e) + \frac{m_e^2}{m_p^2}\sigma_T n_p \rho_\gamma(\vec{v}_\gamma - \vec{v}_p) - \frac{m_p n_p}{a}\nabla\Phi$$

$$m_e n_e\left[\partial_t\vec{v}_e + H\vec{v}_e + \left(\vec{v}_e\cdot\frac{\nabla}{a}\right)\vec{v}_e\right]$$

Coulomb Thomson

$$= -e n_e(\vec{E} + \vec{v}_e\times\vec{B}) + e^2 n_p n_e \eta(\vec{v}_p - \vec{v}_e) + \sigma_T n_e \rho_\gamma(\vec{v}_\gamma - \vec{v}_e) - \frac{m_e n_e}{a}\nabla\Phi$$

simplifying EOMs

relative and center-of-mass quantities

$$n_b \equiv \frac{m_p n_p + m_e n_e}{m_p + m_e}, \qquad \delta n_{pe} \equiv n_p - n_e,$$

$$\vec{v}_b \equiv \frac{m_p n_p \vec{v}_p + m_e n_e \vec{v}_e}{m_p n_p + m_e n_e}, \qquad \delta \vec{v}_{pe} \equiv \vec{v}_p - \vec{v}_e,$$

cosmological perturbations up to 2nd order

$$\rho_\gamma(t, \vec{x}) = \rho_\gamma^{(0)}(t) + \rho_\gamma^{(1)}(t, \vec{x}) + \rho_\gamma^{(2)}(t, \vec{x}) + \cdots ,$$

$$n_b(t, \vec{x}) = n_b^{(0)}(t) + n_b^{(1)}(t, \vec{x}) + n_b^{(2)}(t, \vec{x}) + \cdots ,$$

$$\vec{V}(t, \vec{x}) = \vec{V}^{(1)}(t, \vec{x}) + \vec{V}^{(2)}(t, \vec{x}) + \cdots , \quad \nabla \times \vec{V}^{(1)} = 0$$

$$\vec{B}(t, \vec{x}) = \vec{B}^{(2)}(t, \vec{x}) + \cdots ,$$

→ Hall term and Lorentz force are higher order.

rewriting EOMs

p–e relative motion, γ , γ –baryon relative motion

$$\frac{m_e}{e}\left[\partial_t\delta\vec{v}_{pe}+H\delta\vec{v}_{pe}+\left(\vec{v}_b\cdot\frac{\nabla}{a}\right)\delta\vec{v}_{pe}+\left(\delta\vec{v}_{pe}\cdot\frac{\nabla}{a}\right)\vec{v}_b\right]$$

$$=\vec{E}-\left(en_b\eta+\frac{\sigma_T\rho_\gamma}{e}\right)\delta\vec{v}_{pe}-\frac{\sigma_T\rho_\gamma}{e}\delta\vec{v}_{\gamma b}$$

generalized Ohm's law

$$\partial_t\vec{v}_\gamma+H\vec{v}_\gamma+\left(\vec{v}_\gamma\cdot\frac{\nabla}{a}\right)\vec{v}_\gamma$$

$$=-\frac{1}{4a}\frac{\nabla\rho_\gamma}{\rho_\gamma}-\frac{3}{4}\sigma_T n_b\left(\delta\vec{v}_{\gamma b}+\delta\vec{v}_{pe}\right)-\frac{1}{a}\nabla\Phi$$

$$\partial_t\delta\vec{v}_{\gamma b}+H\delta\vec{v}_{\gamma b}+\left(\vec{v}\cdot\frac{\nabla}{a}\right)\delta\vec{v}_{\gamma b}+\left(\delta\vec{v}_{\gamma b}\cdot\frac{\nabla}{a}\right)\vec{v}-\left(\delta\vec{v}_{\gamma b}\cdot\frac{\nabla}{a}\right)\delta\vec{v}_{\gamma b}$$

$$=-\frac{1}{4a}\frac{\nabla\rho_\gamma}{\rho_\gamma}-\frac{\sigma_T\rho_\gamma}{m_p}\left(\delta\vec{v}_{\gamma b}+\delta\vec{v}_{pe}\right)$$

conventional CMB

Conventionally, we deal with photons and baryons.

$$\frac{m_e}{e}\left[\partial_t \delta\vec{v}_{pe} + H\delta\vec{v}_{pe} + \left(\vec{v}_b\cdot\frac{\nabla}{a}\right)\delta\vec{v}_{pe} + \left(\delta\vec{v}_{pe}\cdot\frac{\nabla}{a}\right)\vec{v}_b\right]$$

$$= \vec{E} - \left(en_b\eta + \frac{\sigma_T\rho_\gamma}{e}\right)\delta\vec{v}_{pe} - \frac{\sigma_T\rho_\gamma}{e}\delta\vec{v}_{\gamma b}$$

$$\partial_t\vec{v}_\gamma + H\vec{v}_\gamma + \left(\vec{v}_\gamma\cdot\frac{\nabla}{a}\right)\vec{v}_\gamma$$

$$= -\frac{1}{4a}\frac{\nabla\rho_\gamma}{\rho_\gamma} - \frac{3}{4}\sigma_T n_b\left(\delta\vec{v}_{\gamma b} + \delta\vec{v}_{pe}\right) - \frac{1}{a}\nabla\Phi$$

$$\partial_t\delta\vec{v}_{\gamma b} + H\delta\vec{v}_{\gamma b} + \left(\vec{v}\cdot\frac{\nabla}{a}\right)\delta\vec{v}_{\gamma b} + \left(\delta\vec{v}_{\gamma b}\cdot\frac{\nabla}{a}\right)\vec{v} - \left(\delta\vec{v}_{\gamma b}\cdot\frac{\nabla}{a}\right)\delta\vec{v}_{\gamma b}$$

$$= -\frac{1}{4a}\frac{\nabla\rho_\gamma}{\rho_\gamma} - \frac{\sigma_T\rho_\gamma}{m_p}\left(\delta\vec{v}_{\gamma b} + \delta\vec{v}_{pe}\right)$$

Solving Maxwell + Ohm

basic equations

- EOM of photons
- EOM of relative motion between photons and baryons
- generalized Ohm's law
- Maxwell equations

} solve in this section (generalized Ohm's law and Maxwell equations)

how to solve

- up to 2nd order in cosmological perturbations
- regard Thomson term as an external source

  $\rightarrow$ Electric charge, current and EM fields are expressed as functions of Thomson term.

basic equations

$$\frac{1}{a}\nabla\cdot\vec{E} = e\delta n_{pe}$$

$$\partial_t\vec{E} + 2H\vec{E} = \frac{1}{a}\nabla\times\vec{B} - e(n_b\delta\vec{v}_{pe} + \delta n_{pe}\vec{v}_b),$$

$$\partial_t\vec{B} + 2H\vec{B} = -\frac{1}{a}\nabla\times\vec{E}$$

$$\partial_t\delta n_{pe} + 3H\delta n_{pe} + \frac{1}{a}\nabla\cdot(n_b\delta\vec{v}_{pe} + \delta n_{pe}\vec{v}_b) = 0$$

EOM of Vpe
↓
Ohm's law

$$\vec{E} = \frac{m_e}{e(1+\beta)}\left[\partial_t\delta\vec{v}_{pe} + \left(\vec{v}_b\cdot\frac{\nabla}{a}\right)\delta\vec{v}_{pe} + \left(\delta\vec{v}_{pe}\cdot\frac{\nabla}{a}\right)\vec{v}_b\right] + en_b\eta_{\rm eff}\delta\vec{v}_{pe} + \vec{C}$$

Thomson term (source)

$$\vec{C} = \frac{\sigma_T\rho_\gamma}{e}\delta\vec{v}_{\gamma b}$$

effective resistivity

$$\eta_{\rm eff} = \eta\left(1 + \frac{\tau_{\rm C}}{\tau_{\rm T}} + H\tau_{\rm C}\right)$$

first order in CP

B = 0 at the 1st order in CP.

$$\frac{1}{a}\nabla\cdot\vec{E}^{(1)} = e\delta n_{pe}^{(1)},$$

$$\partial_t\vec{E}^{(1)} + 2H\vec{E}^{(1)} = -en_b^{(0)}\delta\vec{v}_{pe}^{(1)},$$

$$\partial_t\delta n_{pe}^{(1)} + 3H\delta n_{pe}^{(1)} + \frac{n_b^{(0)}}{a}\nabla\cdot\delta\vec{v}_{pe}^{(1)} = 0,$$

$$\vec{E}^{(1)} = \frac{m_e}{e(1+\beta)}\partial_t\delta\vec{v}_{pe}^{(1)} + en_b^{(0)}\eta_{\rm eff}^{(0)}\delta\vec{v}_{pe}^{(1)} + \vec{C}^{(1)}.$$

First, take the divergence of the Ohm's law.

charge density

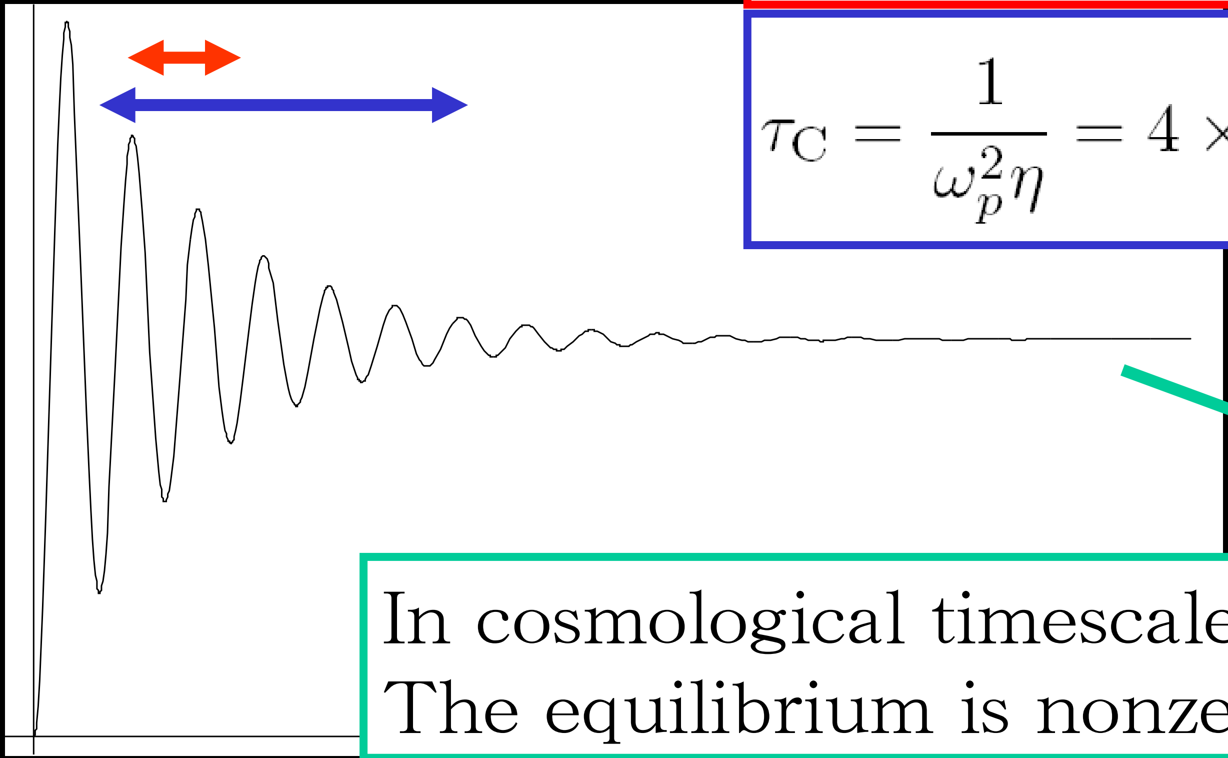
divergence of the generalized Ohm's law

$$\frac{1}{\omega_p^2}\partial_t^2\rho^{(1)} + \eta_{\rm eff}^{(0)}\partial_t\rho^{(1)} + \rho^{(1)} = \frac{1}{a}\nabla\cdot\vec{C}^{(1)}$$

damped oscillation with a source

$$\omega_p^{-1} \equiv \sqrt{\frac{m_e}{e^2 n^{(0)}}} = 2\times 10^{-9}\ \mathrm{sec}\left(\frac{1+z}{10^5}\right)^{-3/2}$$

$$\tau_{\rm C} = \frac{1}{\omega_p^2\eta} = 4\times 10^{-3}\ \mathrm{sec}\left(\frac{1+z}{10^5}\right)^{-3/2}$$



$$\rho^{(1)} = \nabla\cdot\vec{C}^{(1)}$$

In cosmological timescale, plasma oscillation damps.
The equilibrium is nonzero due to the source.

solutions for the 1st order in CP

$$\delta n_{pe}^{(1)} = \frac{1}{ea}\nabla \cdot \vec{C}^{(1)}$$

$$\delta \vec{v}_{pe}^{(1)} = -\frac{1}{e n_b^{(0)}}(\partial_t + 2H)\vec{C}^{(1)}$$

$$\vec{E}^{(1)} = \vec{C}^{(1)},$$

$$\rho^{(1)} = \frac{1}{a}\nabla \cdot \vec{C}^{(1)}$$

$$\vec{j}^{(1)} = -(\partial_t + 2H)\vec{C}^{(1)}$$

Electric charge, current and E field are expressed by the Thomson term.

second order in CP

B field joins at the second order.

$$\nabla \cdot \vec{E}^{(2)} = e\delta n_{pe}^{(2)},$$

$$\partial_t \vec{E}^{(2)} = \nabla \times \vec{B}^{(2)} - e\left(n_b^{(0)}\delta\vec{v}_{pe}^{(2)} + n_b^{(1)}\delta\vec{v}_{pe}^{(1)} + \delta n_{pe}^{(1)}\vec{v}_b^{(1)}\right),$$

$$\partial_t \vec{B}^{(2)} = -\nabla \times \vec{E}^{(2)},$$

$$\partial_t \delta n_{pe}^{(2)} + \nabla \cdot \left(n_b^{(0)}\delta\vec{v}_{pe}^{(2)} + n_b^{(1)}\delta\vec{v}_{pe}^{(1)} + \delta n_{pe}^{(1)}\vec{v}_b^{(1)}\right) = 0,$$

$$\vec{E}^{(2)} = \frac{m_e}{e(1+\beta)}\left[\partial_t\delta\vec{v}_{pe}^{(2)} + \left(\vec{v}_b^{(1)}\cdot\nabla\right)\delta\vec{v}_{pe}^{(1)} + \left(\delta\vec{v}_{pe}^{(1)}\cdot\nabla\right)\vec{v}_b^{(1)}\right]$$

$$+ e n_b^{(0)}\eta_{\rm eff}^{(0)}\left(\delta\vec{v}_{pe}^{(2)} + \frac{n_b^{(1)}}{n_b^{(0)}}\delta\vec{v}_{pe}^{(1)} + \frac{\eta_{\rm eff}^{(1)}}{\eta_{\rm eff}^{(0)}}\delta\vec{v}_{pe}^{(1)}\right) + \vec{C}^{(2)}.$$

solutions up to second order in CP

$$\vec{E} = \vec{C},$$

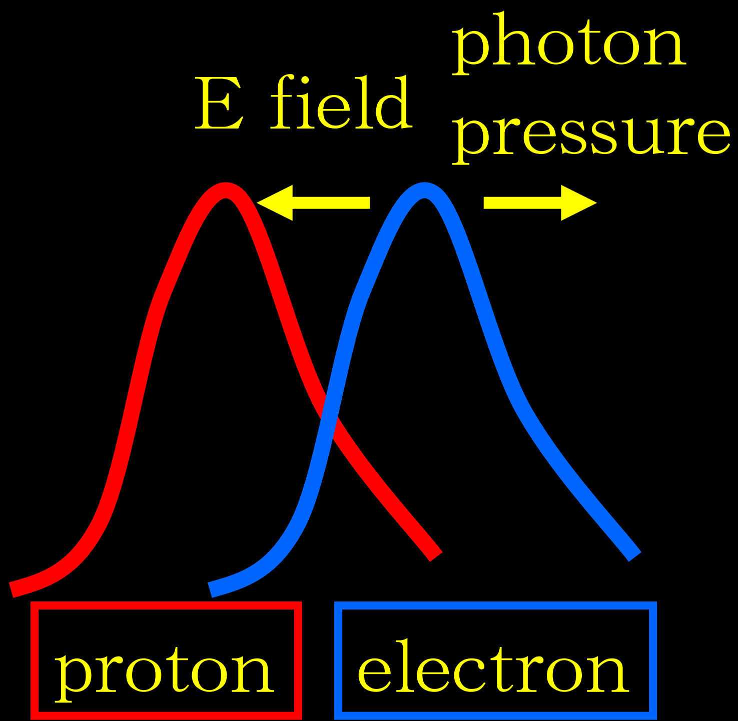
$$\vec{B} = -\frac{1}{a^2}\int dt\ a\nabla \times \vec{C},$$

$$\rho = \frac{1}{a}\nabla \cdot \vec{C},$$

$$\vec{j} = -(\partial_t + 2H)\vec{C} - \frac{1}{a^3}\int dt\ a\nabla \times \nabla \times \vec{C}$$

zero at the 1st order

interpretation



$$\vec{E} = \vec{C},$$

$$\vec{B} = -\frac{1}{a^2}\int dt\ a\nabla \times \vec{C},$$

$$\rho = \frac{1}{a}\nabla \cdot \vec{C},$$

$$\vec{j} = -(\partial_t + 2H)\vec{C} - \frac{1}{a^3}\int dt\ a\nabla \times \nabla \times \vec{C}$$

- electric current term is not important in Ohm's law
  $\rightarrow$ photon pressure balances with E field
- current $\rightarrow$ (displacement current) + (B field)
- E and charge vanish when Thomson term disappear.
  B and current do not because they are integral.

summary of section 2

solving Maxwell + Ohm

- up to second order in CP
- Thomson term was treated as an external source
  → Electromagnetic quantities are expressed
  by the Thomson term.
- We need the Thomson term (photon-baryon relative velocity) to evaluate B.

3. Magnetogenesis & tight coupling approximation

magnetogenesis

B field and Thomson term

$$\vec{B} = -\frac{1}{a^2}\int dt\ a\nabla \times \vec{C}$$

$$\vec{C} = \frac{\sigma_T \rho_\gamma}{e}\delta\vec{v}_{\gamma b}$$

vector product of density gradient of photons and velocity difference

$$\nabla \times \vec{C}^{(2)} = \frac{\sigma_T \rho_\gamma^{(0)}}{e}\left[\frac{\nabla \rho_\gamma^{(1)}}{\rho_\gamma^{(0)}} \times \delta\vec{v}_{\gamma b}^{(1)} + \nabla \times \delta\vec{v}_{\gamma b}^{(2)}\right]$$

vorticity difference

We solve for δ v by tight coupling approximation.
(Peebles & Yu, 1970; Kobayashi, Maartens, Shiromizu & KT, 2007)

EOM for γ –b relative motion

$$\partial_t \delta\vec{v}_{\gamma b} + H\delta\vec{v}_{\gamma b} + \left(\vec{v}\cdot\frac{\nabla}{a}\right)\delta\vec{v}_{\gamma b} + \left(\delta\vec{v}_{\gamma b}\cdot\frac{\nabla}{a}\right)\vec{v} - \left(\delta\vec{v}_{\gamma b}\cdot\frac{\nabla}{a}\right)\delta\vec{v}_{\gamma b}$$
$$= -\frac{1}{4a}\frac{\nabla\rho_\gamma}{\rho_\gamma} - \frac{\sigma_T\rho_\gamma}{m_p}\left(\delta\vec{v}_{\gamma b} + \delta\vec{v}_{pe}\right)$$

Electric current has a negligible effect on the dynamics of the γ –b relative motion.

$$\frac{|\delta\vec{v}_{pe}|}{|\delta\vec{v}_{\gamma b}|} \sim \frac{\sigma_T\rho_\gamma k}{e^2 a n_b} = \frac{k}{a\omega_p^2\tau_{\mathrm{T}}} \sim 1.5\times 10^{-27}\left(\frac{k}{aH}\right)\left(\frac{1+z}{10^5}\right)^3$$

tight coupling approximation (TCA)

$$\frac{m_p}{\sigma_T\rho_\gamma}\frac{k}{a} = 2.4\times 10^{-6}\left(\frac{k}{aH}\right)\left(\frac{1+z}{10^5}\right)^{-2}$$

Time derivative is less important.

$$\delta\vec{v}_{\gamma b} = \delta\vec{v}_{\gamma b}^{(I)} + \delta\vec{v}_{\gamma b}^{(II)} + \cdots$$

TCA I

(I,1) → TCA I & CP 1, (I,2) → TCA I & CP 2

$$\delta\vec{v}_{\gamma b}^{(I,1)} = -\frac{1}{4}\frac{m_p}{\sigma_T \bar{\rho}_\gamma^{(0)}}\frac{\nabla\bar{\rho}_\gamma^{(1)}}{\bar{\rho}_\gamma^{(0)}},$$

$$\delta\vec{v}_{\gamma b}^{(I,2)} = -\frac{1}{4}\frac{m_p}{\sigma_T \bar{\rho}_\gamma^{(0)}}\left[\frac{\nabla\bar{\rho}_\gamma^{(2)}}{\bar{\rho}_\gamma^{(0)}} - 2\frac{\bar{\rho}_\gamma^{(1)}}{\bar{\rho}_\gamma^{(0)}}\frac{\nabla\bar{\rho}_\gamma^{(1)}}{\bar{\rho}_\gamma^{(0)}}\right]$$

However, the source of B is zero at TCA I.

$$\nabla \times \vec{C}^{(2)} = \frac{\sigma_T \rho_\gamma^{(0)}}{e}\left[\frac{\nabla\rho_\gamma^{(1)}}{\rho_\gamma^{(0)}} \times \delta\vec{v}_{\gamma b}^{(1)} + \nabla \times \delta\vec{v}_{\gamma b}^{(2)}\right]$$

We must go on to TCA II.

TCA II

$$\delta\vec{v}_{\gamma b}^{(II,1)} = -\frac{1}{4\bar{\nu}^{(0)}}\nabla\Delta_\gamma^{(I,1)} + \frac{1}{4(\bar{\nu}^{(0)})^2}\frac{\partial_t\nabla\bar{\rho}_\gamma^{(1)}}{\bar{\rho}_\gamma^{(0)}}$$

This is not parallel to $\nabla\rho$ in general.

$$\begin{aligned}\delta\vec{v}_{\gamma b}^{(II,2)} = & -\frac{1}{4\bar{\nu}^{(0)}}\left[\nabla\Delta_\gamma^{(I,2)} - \frac{\bar{\nu}^{(1)}}{\bar{\nu}^{(0)}}\Delta_\gamma^{(I,1)} - \frac{\nu^{(I,1)}}{\bar{\nu}^{(0)}}\frac{\nabla\bar{\rho}_\gamma^{(1)}}{\bar{\rho}_\gamma^{(0)}}\right] \\ & + \frac{1}{4(\bar{\nu}^{(0)})^2}\left[\frac{\partial_t\nabla\bar{\rho}_\gamma^{(2)}}{\bar{\rho}_\gamma^{(0)}} - \left(\frac{\bar{\rho}_\gamma^{(1)}}{\bar{\rho}_\gamma^{(0)}} + \frac{2\bar{\nu}^{(1)}}{\bar{\nu}^{(0)}}\right)\frac{\partial_t\nabla\bar{\rho}_\gamma^{(1)}}{\bar{\rho}_\gamma^{(0)}} - \left(\frac{\partial_t\bar{\rho}_\gamma^{(1)}}{\bar{\rho}_\gamma^{(0)}} + \frac{\partial_t\bar{\nu}^{(1)}}{\bar{\nu}^{(0)}}\right)\frac{\nabla\bar{\rho}_\gamma^{(1)}}{\bar{\rho}_\gamma^{(0)}} \right. \\ & \left. + (\vec{v}\cdot\nabla)\frac{\nabla\bar{\rho}_\gamma^{(1)}}{\bar{\rho}_\gamma^{(0)}} + \left(\frac{\nabla\bar{\rho}_\gamma^{(1)}}{\bar{\rho}_\gamma^{(0)}}\cdot\nabla\right)\vec{v}\right],\end{aligned}$$

B is generated at TCA II.

results

$$\rho^{(1)} = -\frac{1}{4}\frac{m_p}{e}\frac{\nabla^2\bar{\rho}_\gamma^{(1)}}{\bar{\rho}_\gamma^{(0)}},$$

$$\vec{j}^{(1)} = \frac{1}{4}\frac{m_p}{e}\frac{\partial_t\nabla\bar{\rho}_\gamma^{(1)}}{\bar{\rho}_\gamma^{(0)}},$$

$$\vec{E}^{(1)} = -\frac{1}{4}\frac{m_p}{e}\frac{\nabla\bar{\rho}_\gamma^{(1)}}{\bar{\rho}_\gamma^{(0)}},$$

$$\vec{B}^{(2)} = -\frac{1}{16}\bar{R}^{(0)}\frac{m_p^2}{e\sigma_T\bar{\rho}_\gamma^{(0)}}\int dt\,\frac{\nabla\bar{\rho}_\gamma^{(1)}}{\bar{\rho}_\gamma^{(0)}}\times\left[\frac{\partial_t\nabla\bar{\rho}_\gamma^{(1)}}{\bar{\rho}_\gamma^{(0)}}+\int dt\,\frac{\nabla(\nabla^2\bar{\rho}_\gamma^{(1)})}{\bar{\rho}_\gamma^{(0)}}\right]$$

All EM quantities are expressed by familiar quantities.

Evaluation of B spectrum is now in progress,
but the anisotropic stress must be included to be complete.

another approach

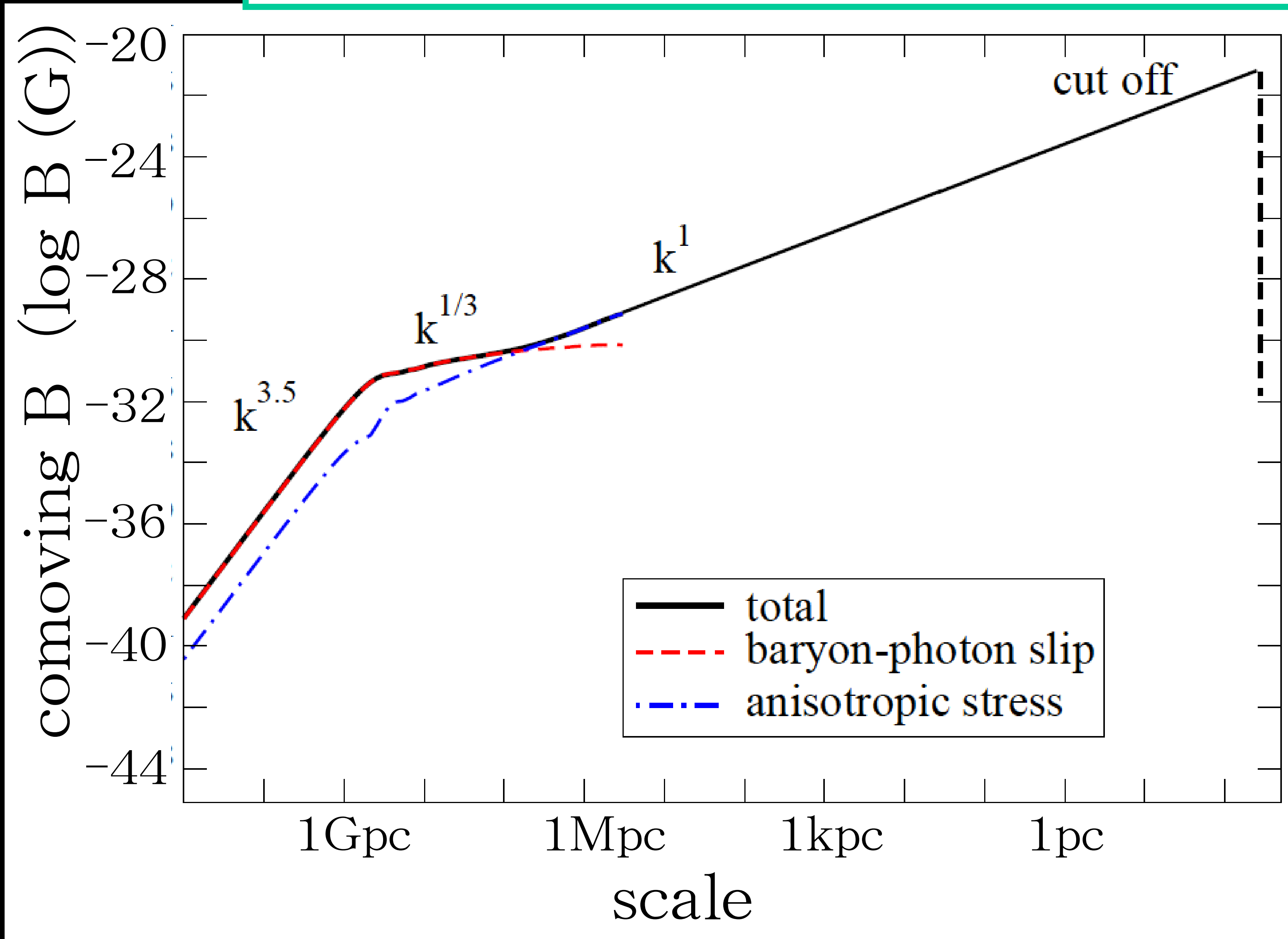
Ichiki, KT et al. (2006)

- derive the source term (relativistic)

$$\partial_t B^i = \frac{8\sigma_T \rho_\gamma^{(0)}}{3e}\epsilon^{ijk}\left[\frac{\rho^{(1)}_{\gamma,k}}{\rho_\gamma^{(0)}}\delta v^{(1)}_{b\gamma\ j} + \delta v^{(2)}_{b\gamma\ j,k} + \frac{1}{8}\left(v^{(1)}_{b\ l}\Pi^{(1)l}_{\gamma\ j}\right)_{,k}\right]$$

- numerically calculate the spectrum contributed from (1st order) × (1st order), neglecting the vorticity (purely 2nd order) spectrum

$$\partial_t B^i = \frac{8\sigma_T \rho_\gamma^{(0)}}{3e} \epsilon^{ijk} \left[\frac{\rho_{\gamma,k}^{(1)}}{\rho_\gamma^{(0)}} \delta v_{b\gamma\ j}^{(1)} + \delta v_{b\gamma\ j,k}^{(2)} + \frac{1}{8} \left(v_{b\ l}^{(1)} \Pi_{\gamma\ j}^{(1)l} \right)_{,k} \right]$$



toward the complete evaluation

numerical approach

- numerically cancel TCA I
- include vorticity (purely 2nd order)
- confirm the validity of TCA

TCA approach

- TCA I already canceled by hand
- include anisotropic stress

These two approaches are necessary
to cross check and evaluate the spectrum.

3 components

proton, electron → { electric current, baryon }

baryon, photon → { photon – baryon center of mass }

photon

timescales

$$\tau_{\mathrm{C}} = \frac{1}{\omega_p^2 \eta} = 4 \times 10^{-3} \ \mathrm{sec} \left(\frac{1+z}{10^5} \right)^{-\frac{3}{2}}$$

$$\tau_{\mathrm{T}} = \frac{m_p}{\sigma_T \rho_\gamma} = 10^3 \ \mathrm{sec} \left(\frac{1+z}{10^5} \right)^{-4}$$

$$H^{-1} = 4.5 \times 10^9 \ \mathrm{sec} \left(\frac{1+z}{10^5} \right)^{-2}$$

Because $H\,\tau \ll 1$, we will use tight coupling approximation.

tight coupling approximation (TCA)

We need the velocity difference.

$$\partial_t \delta\vec{v} = -\frac{1}{\tau}\delta\vec{v} + \vec{A}$$

If (scattering time) $\ll$ (dynamical timescale)

$$\tau\partial_t \sim \tau H \ll 1$$

tight coupling expansion is good. (Peebles & Yu, 1970)

$$\delta\vec{v} = \delta\vec{v}^{(I)} + \delta\vec{v}^{(II)} + \cdots \qquad \vec{A} = \vec{A}^{(0)} + \vec{A}^{(I)} + \cdots$$

TCA I

$$0 = -\frac{1}{\tau}\delta\vec{v}^{(I)} + \vec{A}^{(0)}$$

TCA II

$$\partial_t \delta\vec{v}^{(I)} = -\frac{1}{\tau}\delta\vec{v}^{(II)} + \vec{A}^{(I)}$$

$$\left|\delta\vec{v}^{(I)}\right| \sim H\tau v$$

$$\left|\delta\vec{v}^{(II)}\right| \sim (H\tau)^2 v$$

EOM of photon–baryon relative motion

$$\partial_t \delta\vec{v}_{\gamma b} + H\delta\vec{v}_{\gamma b} + \left(\vec{v}\cdot\frac{\nabla}{a}\right)\delta\vec{v}_{\gamma b} + \left(\delta\vec{v}_{\gamma b}\cdot\frac{\nabla}{a}\right)\vec{v} - \frac{1-R}{1+R}\left(\delta\vec{v}_{\gamma b}\cdot\frac{\nabla}{a}\right)\delta\vec{v}_{\gamma b}$$
$$= -\frac{1}{4a}\frac{\nabla\rho_\gamma}{\rho_\gamma} - \frac{1+R}{1+\beta}\frac{\sigma_T\rho_\gamma}{m_p}\left[(1+\beta^2)\delta\vec{v}_{\gamma b} + \underline{\frac{1-\beta^3}{1+\beta}\delta\vec{v}_{pe}}\right].$$

contribution from electric current

$$\frac{|\delta\vec{v}_{pe}|}{|\delta\vec{v}_{\gamma b}|} \sim \frac{\sigma_T\rho_\gamma k}{e^2 a n_b} = \frac{k}{a\omega_p^2\tau_{\mathrm{T}}} \sim 1.5\times 10^{-27}\left(\frac{k}{aH}\right)\left(\frac{1+z}{10^5}\right)^3$$

Electric current is negligible.

Protons and electrons can be treated as one fluid.

order estimation

deviation between photons and baryons

$$\Delta^{(I,1)} \sim \left|\delta\vec{v}_{\gamma b}^{(1)}\right| \sim \frac{1}{4}\frac{m_p}{\sigma_T\rho_\gamma^{(0)}}\frac{k^2}{a^2H}\delta_\gamma = \frac{1}{4}\frac{k^2\tau_{\rm T}}{a^2H}\delta_\gamma$$

$$\sim 7\times10^{-13}\left(\frac{k}{aH}\right)^2\left(\frac{1+z}{10^5}\right)^{-1}$$

deviation between protons and electrons

$$\left|\frac{\rho^{(1)}}{en_b^{(0)}}\right| \sim \left|\delta\vec{v}_{pe}^{(1)}\right| \sim \frac{m_p}{4e^2n_b^{(0)}}\frac{k^2}{a^2}\delta_\gamma = \frac{1}{4\beta}\frac{k^2}{a^2\omega_p^2}\delta_\gamma$$

$$\sim 9\times10^{-40}\left(\frac{k}{aH}\right)^2\left(\frac{1+z}{10^5}\right)^3$$